Work & Energy

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Introduction to Work & Energy

Introduction to Work & Energy

Work and energy are two sides of the same coin, so to speak. Everyone always wants to say energy is …, or work is …, the dots being a multitude of very unsatisfying statements. The problem is that it is somewhat of a nebulous concept, hence our unease.

I typically ask my students, half-jokingly, for the meaning of life when we get to this unit. Not just human life, but any life – plant, or animal, or otherwise. Every once in a while, I get the answer I am looking for – probably because they find it humorous. The meaning of life is to eat – so that we (plant, animal, or otherwise) will have enough energy to do whatever needs to be done to find our next meal (by hunting, growing, …) – and round and round we go. Of course if there is enough energy available then other things can then be done – reproduction, grooming, creation of music, etc.

The point is that when work is done to a system, it gains energy; when work is done by the system, it loses energy. That energy can be used to do work, it can transform into other forms of energy because of work. But because of non-conservative forces (think friction resulting in heat) we can't really undo a process fully. (Ultimately it ties into entropy – hence the unease.) Energy is something that can be quantified, and its movement in and out of a system and what happens to it within the system as a result of the work that is done can be tracked, and the total amount is conserved.

So, the best we can do is start with work: $dW_F \equiv \vec{F} \cdot d\vec{x}$ or $W_F \equiv \int \vec{F}(\vec{x}) \cdot d\vec{x}$, and then define energy as the value of that quantity. Thus, work is the mechanism by which energy enters, leaves, or moves within a system.

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Introduction to Work & Energy

Introduction to Work & Energy

Forms of Work and Energy:

Mechanical
- due to motion or position of macroscopic particles

Chemical
- due to motion or position of electrons in atoms or molecules

Nuclear
- due to motion or position of protons and neutrons in the nuclei of atoms

Heat
- due to motion of microscopic particles in bulk objects

Electromagnetic
- due to motion of photons

Notice the pattern – most have to do with both motion and position of physical objects
- Motion
  - Kinetic Energy
- Position
  - Potential Energy

Exceptions
- Electromagnetic, because photons are massless.
- Heat, because it is a result of non-conservative forces

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Work

Energy

Definitions:

Kinetic Energy – Energy due to Motion

- $KE \equiv \frac{1}{2}mv^2$ for a mass m with speed v
- Sometimes indicated by T rather than KE
- Scalar quantity
  - Can be associated with different directions, but it is not a vector, and can be translational and/or rotational
    - ✓ $KE_{total} = KE_x + KE_y + KE_{rot}$
    - ✓ $= \frac{1}{2}m(v_x)^2 + \frac{1}{2}m(v_y)^2 + \frac{1}{2}I\omega^2$

Potential Energy – Energy due to position

- Position can be position of point object, or position of atoms that make up object (deformation of object that is a spring)
- Indicated by PE and sometimes U
- Scalar quantity
- $PE_{mg} = mgh$
  - Close to surface of celestial object with gravitational field g
- $U_g = -G\frac{m_1 m_2}{r}$
  - Gravitational potential energy between 2 masses a distance r apart
- $PE_{spring} = \frac{1}{2}kx^2$
  - For spring obeying Hooke's Law
- $PE_f \equiv -\int_{initial}^{final} \vec{F}(\vec{x}) \cdot d\vec{x}$ iff $\vec{F}$ is a conservative force
  - $PE = 0$, the zero of potential energy, generally corresponds to the initial position in the integral above

Power – rate at which work is done

- $P_F = \frac{W_F}{t} = \frac{\vec{F}\cdot\vec{d}}{t} = \vec{F}\cdot\vec{v}$ if $\vec{v}$ is constant

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Potential/Kinetic Energy Worksheet

1. What is the kinetic energy of a $100\ kg$ man having a speed of $2\frac{m}{s}$?

2. What is the kinetic energy of a $50\ g$ $(0.05\ kg)$ bullet moving at $100\ \frac{m}{s}$?

3. What is the kinetic energy of a $3000\ kg$ car at rest?

4. What is the speed of a $3000\ kg$ car having a kinetic energy of $600,000\ J$?

5. What is the mass of a bird having a kinetic energy of $2000\ J$ and a speed of $2\ \frac{m}{s}$?

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Potential/Kinetic Energy Worksheet page 1

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Potential/Kinetic Energy Worksheet

6. What is the gravitational potential energy of a $100\ kg$ woman standing on the ground?

7. What is the gravitational potential energy of a $100\ kg$ woman standing on the roof of a $16\ m$ tall building?

8. What is the potential energy of a 747 ($170\,000\ kg$) flying at an altitude of $10\ km$ ($10,000\ m$) and with a speed of $900\ \frac{km}{hr}$?

9. What is the kinetic energy of a 747 (170 000 kg) flying at an altitude of 10 km and with a speed of $900\ \frac{km}{hr}$ ($250\ \frac{m}{s}$)?

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Potential/Kinetic Energy Worksheet page 2

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Work

Energy

Definitions:

Work done by a force $\vec{F}$ following the path from the initial position to the final position:

$$W_F \equiv \int_{initial}^{final} \vec{F}(\vec{x}) \cdot d\vec{x}$$

For a non-calculus based class, this truncates to

$$W_F = \vec{F} \cdot \vec{d} = Fd\cos\theta = Fd_{\parallel} = F_{\parallel}d$$

where W_F is the work done by the force F, d is the displacement of the object on which the force acts, and θ is the angle between the direction of the force and the direction of the displacement, $F_{\parallel}$ is the component of force in the direction of the motion, and $d_{\parallel}$ is the component of the displacement in the direction of the force. This all assumes that the force is constant and the motion is one-dimensional.

If the force F is a conservative force, then a potential energy can be defined. The potential energy is the negative of the work done by the force when the object moves from the reference point to its final position.

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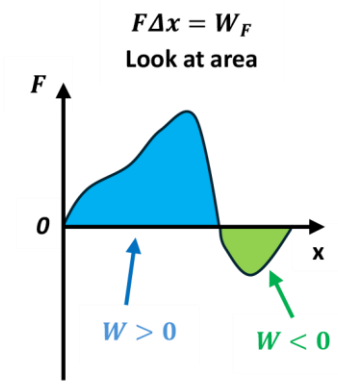
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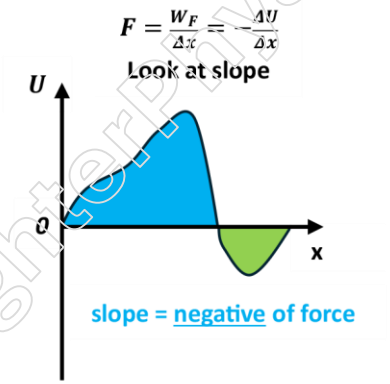
Work

Energy

Force-Position Graphs

As with all other graphs, we can try to interpret the slope and/or the area under the graph for one-dimensional problems.





The area under the force-position graph is the work done by the force.

If force is a conservative force, a potential energy can be defined, which is the negative of the work done by the force. In this case, the force is the negative of the slope of the potential energy-position graph. If the force is not conservative, then the work depends on the path, and therefore the potential energy cannot be defined. This second graph is beyond the scope of AP1, but not for APC. It is here for completeness.

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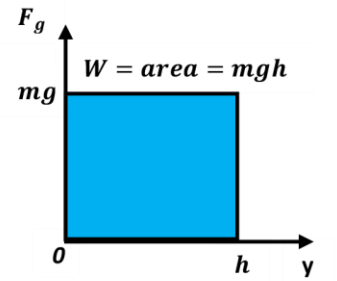
Work

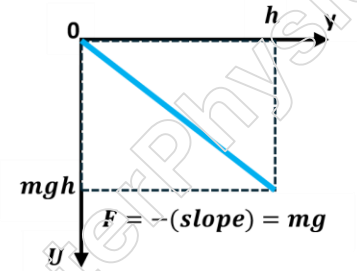
Energy

Force-Position Graphs

Gravitational force:

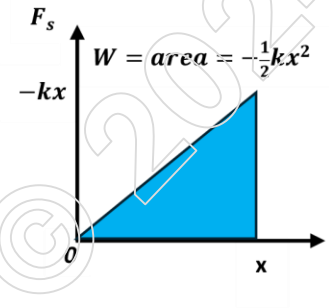
A mass, starting at the origin, falls a distance h.
Take potential energy of zero at the origin.

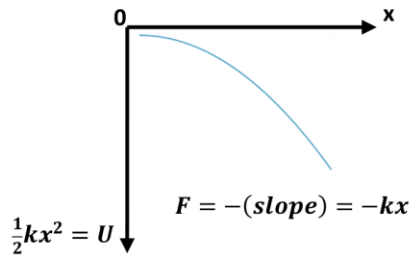




Spring force:

Starting at the origin, a spring is stretched or compressed x.
Take potential energy of zero unstretched.





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General Procedure for Work Calculation

Work Procedure

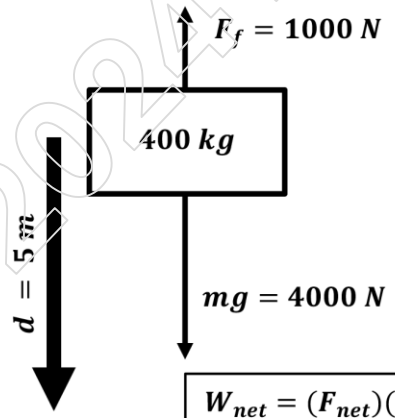
$$W_F = \vec{F} \cdot \vec{d} = Fd\cos\theta$$

- Draw FRD for the system (need forces)
- Draw kinematic diagram for system (need displacement)
- Identify the angle θ from the above diagrams
- Apply to the work equation

Example:

A $400\ kg$ bear slides $5\ m$ down a tree. The tree exerts an upward friction force of $1000\ N$.

There are 2 forces acting. We can find the work done by both forces and the work done by the net force as well.



$$W_{mg} = (mg)(d)(\cos(\theta))$$
$$= (4000)(5)(\cos(0°))$$
$$= 20000\ J$$

$$W_f = (F_f)(d)(\cos(\theta))$$
$$= (1000)(5)(\cos(180°))$$
$$= -5000\ J$$

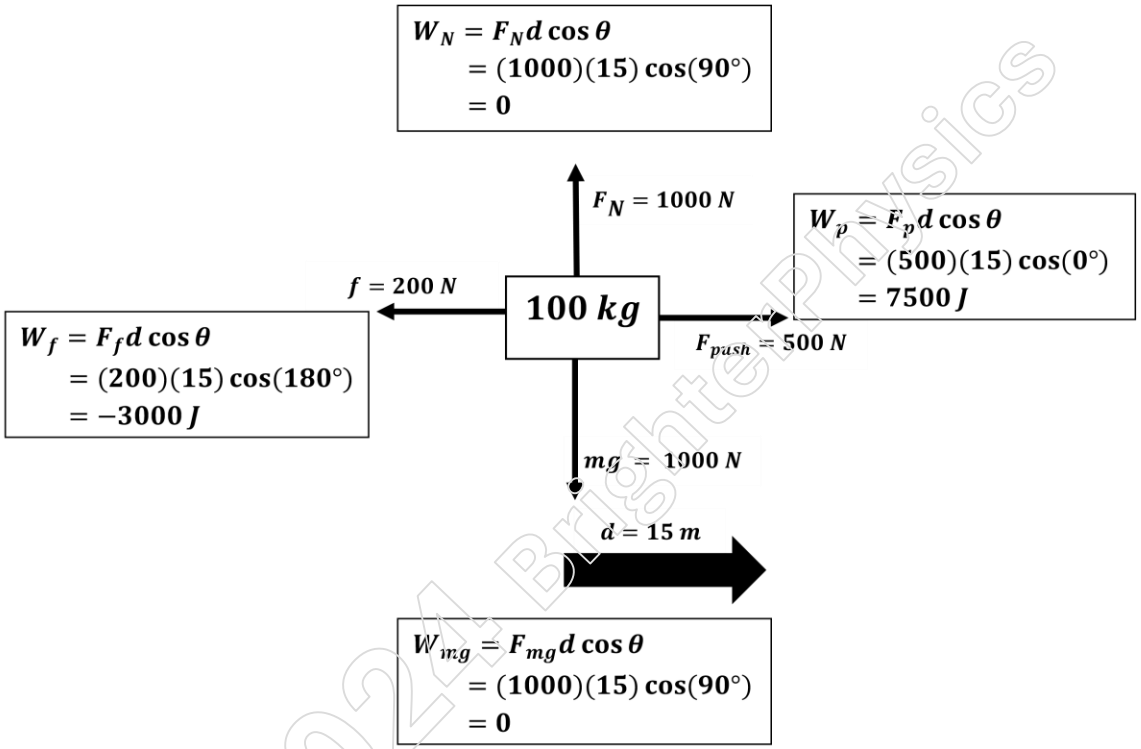
$$W_{net} = (F_{net})(d)(\cos(\theta)) \quad \text{OR} \quad W_{net} = W_{mg} + W_f$$
$$= (3000)(5)(\cos(0°)) \qquad = 20000 - 5000$$
$$= 15000\ J \qquad = 15000\ J$$

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Work Example 1

A $100\ kg$ box on a horizontal table is pushed with $500\ N$ force $15\ m$. The force of Friction between the box and table is $200\ N$.



$F_{net} = 300\ N\ to\ right$

$W_{net} = F_{net} d\cos\theta$
$= (300)(15)\cos(0°)$
$= 4500\ J$

OR

$W_{net} = W_p + W_N + W_f + W_{mg}$
$W_{net} = 7500 + 0 - 3000 + 0$
$= 4500\ J$

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Work Example 2

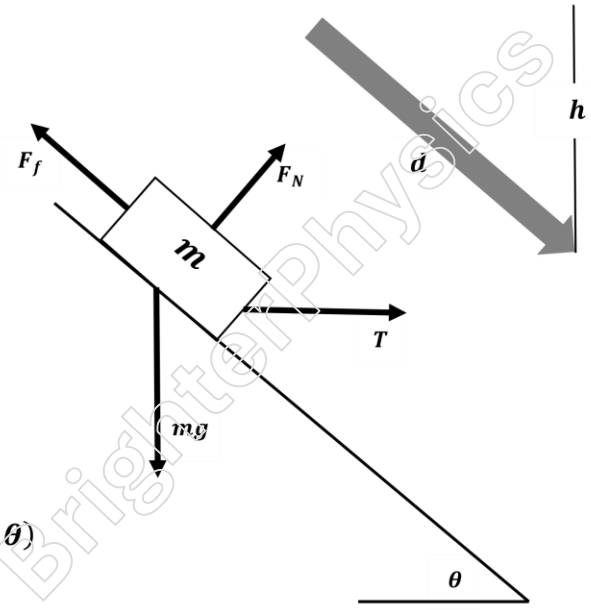
A block of mass m is pulled by a rope with tension T a distance d down a rough ramp that makes an angle of θ from the horizontal.

$$W_T = (T)(d)\cos\theta$$

$$W_N = (N)(d)\cos(90°) = 0$$

$$W_f = (F_f)(d)\cos(180°)$$
$$= -F_f d$$

$$W_{mg} = (mg)(d)\cos(90° - \theta)$$
$$= mgd\sin\theta$$
$$= +mgh$$



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Observations:

Tension: nothing special here

Normal: $W_N = 0$ since F_N is typically perpendicular to the motion

Friction: $W_f < 0$, always negative, since sliding friction always opposes the motion

Gravity: $W_{mg} =$ ±mgh where h is the change in vertical height (compare to PE_{mg})

- **$= +mgh$ if motion is down**
- **$= -mgh$ if motion is up**

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Work Worksheet I

1. A 400 kg bear grasping a vertical tree slides down at constant velocity.
   a) What is the frictional force that acts on the bear?
   b) What is the work done by gravity if the bear slides 10 $meters$?
   c) What is the work done by friction if the bear slides 10 $meters$?
   d) What is the total work done?

2. A 50 kg cart is pushed horizontally 25 $meters$ by a 30 N force.
   a) What is the work done by gravity?
   b) What is the work done by the 30 N force?

3. You slide a 2 kg sled 100 $meters$ across a horizontal surface by pushing it with a horizontal force of 20 N. The force of friction on the sled is 12 N.
   a) What is the work done by gravity?
   b) What is the work done by friction?
   c) What is the work done by the applied 20 N force?

4. How far must you push with a 200 N force to do 5000 J of work?

5. Gravity does 600,000 J of work on a 75 kg object in free fall (let g = 10 m/s^2). How far does this object fall?

6) The kinetic energy of a man is 200 J. If his speed doubles, what is the new kinetic energy?

7) What is the kinetic energy of a speeding bullet is 250 J. What is the kinetic energy of bullet twice as massive and moving three times as fast?

8) The potential energy of a man on the third floor is 10000. What is his potential energy on the ninth floor?

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Work I Worksheet

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Work Worksheet II

1. A 400 kg bear grasping a vertical tree slides down 10 m in 5 sec at constant velocity.
   a) What is the rate at which work is done by gravity on the bear?
   b) What is the power developed by friction?
2. A 50 kg cart is pushed horizontally 25 meters in 12.5 sec by a 30 N force.
   a) What is the power developed by gravity?
   b) What is the power delivered by the 30 N force?
3. You slide a 2 kg sled 100 meters in 25 sec across a horizontal surface by pushing it with a horizontal force of 20 N. The force of friction on the sled is 12 N.
   a) What is the power delivered by gravity?
   b) What is the power delivered by friction?
   c) What is the power delivered by the applied 20 N force?
4. If a 200 N force delivers 5000 W of power, how long must it act to do 200,000 J of work?
5. Your electric bill is expressed in terms of kilowatt-hours (kW·h), a unit of energy delivered by the flow of 1 kilowatt of electricity for 1 hour. How many joules of energy do you get when you buy 1 kW-h of power?
6. A small motor does 4000 J of work in 20 s. What is the power of the motor in watts?
7. A 50 g bullet moving at 100 m/s is stopped by a bullet proof vest in 0.01 s. What is the power delivered by the bullet to the vest?
8. What is the potential energy of a 100 kg rock on top of a 2000 m mountain?
9. Niagara Falls has a height of 80 m.
   a) What is the potential energy of 1 kg of water at the top of the Falls?
   b) If 8 million kg of water goes over the falls per second, what power is delivered to the bottom of the Falls by the falling water.
   c) What is the speed of the water just before it strikes the bottom of the Falls?

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Work II Worksheet

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Work due to mg

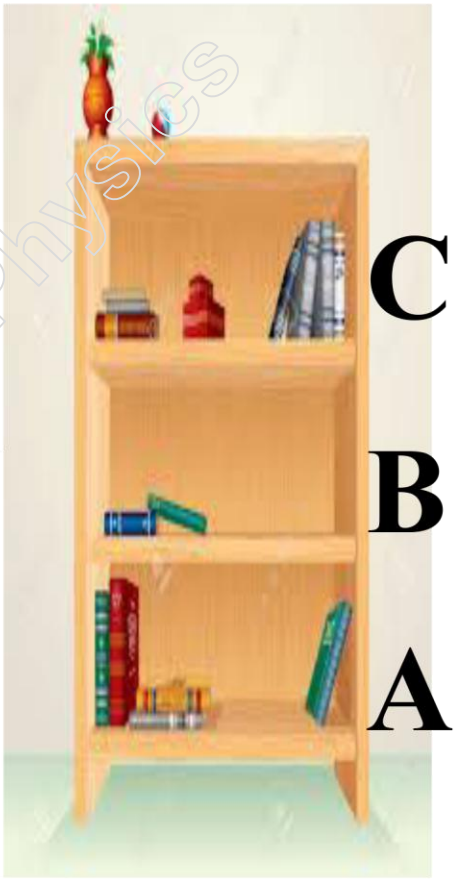
$$W_{mg}$$

$$W_{mg} = \pm mgh$$

Let shelf height be h. Tabletop is level with shelf B.
Find W_{mg} in moving the book from the table to the named shelf?



h



$$W_{\dashrightarrow C} = -mgh$$

$$W_{\dashrightarrow B} = 0$$

$$W_{\dashrightarrow A} = +mgh$$

Note – because gravity is a conservative force, the work done is <u>independent</u> of the path taken to get the book from the table to the shelf!

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Sign of Work

WORK
Positive/Negative/Zero

Positive Work:

- results from force acting in same direction as the motion
- tends to make the speed greater
- tends to increases KE

Negative Work:

- results from force acting in direction opposing the motion
- tends to make the speed slower
- tends to decrease KE

Zero Work:

- result of force acting in direction perpendicular to motion
- tends to not change the speed
- tends to maintain same KE

"Tends" because there may be other forces acting. The change in speed and KE is the cumulative effect of all the forces acting simultaneously.

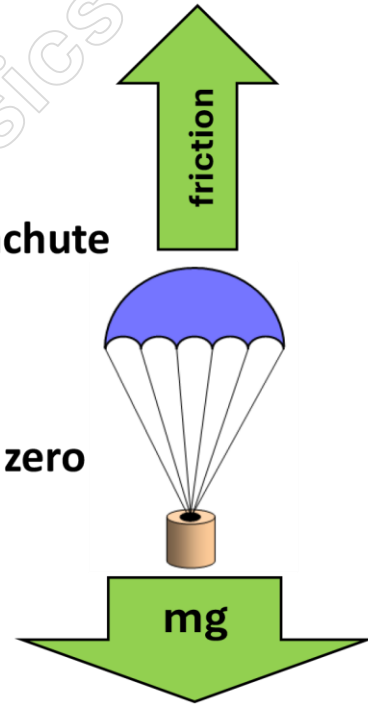
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Sign of Work

WORK
Positive/Negative/Zero

Suppose an object attached to a parachute moving downwards:



W_{mg}:

- positive work is done as mass falls
- mass would speed up if there was no parachute

$W_{friction}$:

- negative work is done as mass falls
- mass would slow down if g was suddenly zero

What happens depends on the relative size of the forces:

$mg > friction$:	$mg < friction$:	$mg = friction$:
$mass\ speeds\ up$	$mass\ slows\ down$	$constant\ speed$
$\Delta KE > 0$	$\Delta KE > 0$	$\Delta KE = 0$
$W_{net} > 0$	$W_{net} > 0$	$W_{net} = 0$

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Sign of Work

WORK
Positive/Negative/Zero
Truck vs. Robot

$W_{robot\ on\ truck} < 0$

since force is exerted up the hill and the truck is moving down the hill



The robot does negative work on the truck

If the truck is speeding up as it goes down the hill,

it will continue to speed up as it rolls over the robot,

<u>BUT</u>

not speeding up as much as it would have had the robot not been there!

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Work-Energy Theorem

Work-Energy Theorem

Arguably, Conservation of Total Energy is the most important principle of Physics:

Total Initial System Energy	+	*Total Work Done*	=	*Total Final System Energy*

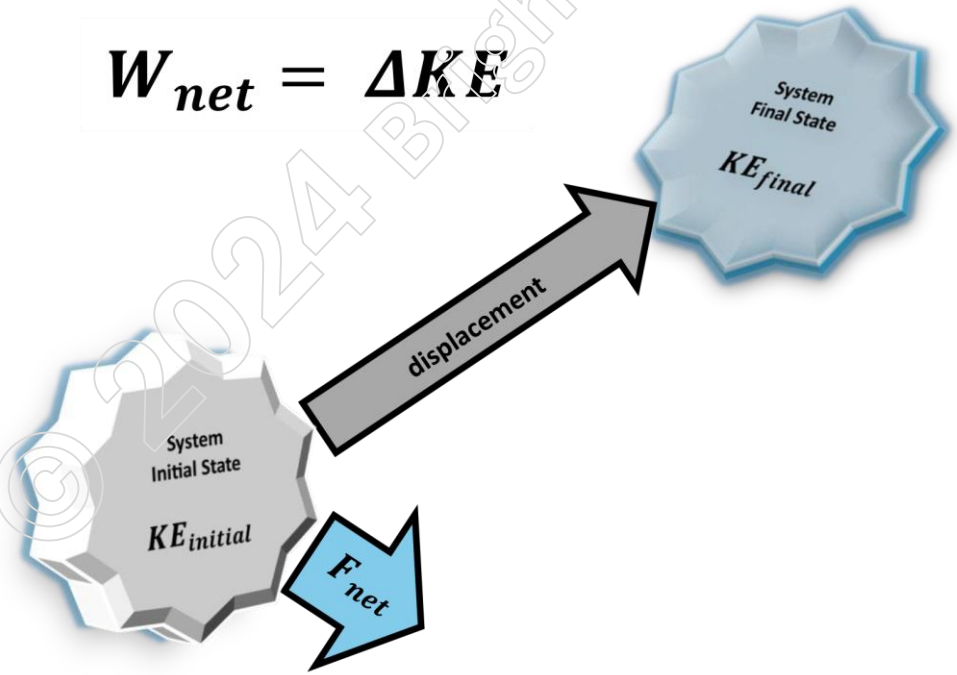
This can be rewritten as:

$$KE_{initial} + F_{net} d \cos\theta = KE_{final}$$
$$KE_{initial} + W_{net} = KE_{final}$$

So finally:

Work-Energy Theorem

$$W_{net} = \Delta KE$$



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Work-Energy Theorem

Work-Energy Theorem

Typically, the Work-Energy Theorem is derived for a system of one object subjected to a constant net force and is very straight forward, which I will not present here.

Here I remove the above restrictions. The system is not limited to a single object, nor a constant net force and is based on scalar equations, not vector equations. This all makes it applicable to any general system.

In this presentation, $SystemEnergy_{initial} = Work + SystemEnergy_{final}$ which is simply Conservation of Total Energy, one must look at the work done by all forces acting on, and within, the system, not just $\vec{F}_{net,external}$. It is true that the internal forces cancel in pairs; however, it is also true that the work done within the system does not necessarily cancel – if two objects within the system rub against each other, the action-reaction pair cancels but the work done by friction is negative for both. [This does not affect the velocity of the center-of-mass, only the kinetic energy of the system.]

The work done is certainly a sum of the work done by all forces, internal and external. If the system consists of a single point mass object, as we and our student typically consider, than there are no internal forces, and the displacement of that one object is the displacement needed to determine the work done by each force acting on the system of one mass. The $SystemEnergy$ of a system consisting of only one point mass is simply the kinetic energy of that mass, and the Work-Energy Theorem is shown.

For a single extended mass object, there are obviously internal forces holding the object together, in which case the $SystemEnergy$ would also include the electrostatic potential energy between the atoms holding the mass together. [To be simplistic about this, this (negative) potential energy is also just equal to twice the kinetic energy of the electrons involved in the bond, assuming classical elliptical atomic orbits.] But this energy is also present in the both the initial and final systems and so cancel if there is no change in internal structure; so again, the Theorem is validated.

If, as a result of the external forces, a strain is created within the single extended mass defining the system, then at the atomic level atoms have changed position relative to their initial positions, and the internal electrostatic force has acted over this displacement and does work. In this case our single object is not immutable as assumed in the simple statement of “a single mass”, and this case is subsumed in the treatment below. It is true that the electrostatic force is conservative and therefore the work done can be quantified in terms of potential energy, but the number of atoms involved, and the resulting number of individual atomic displacements, make the potential energy calculation intractable. We therefore refer to nonconservative forces acting, which is essentially an averaging of the intractable number of conservative interactions over the bulk object.

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Work-Energy Theorem

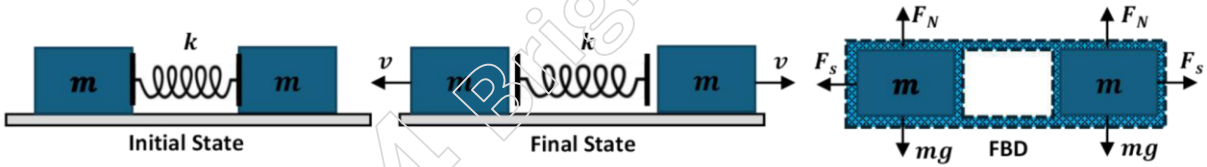
Work-Energy Theorem

The displacement vector in the diagram is the displacement of the center of mass of the system, not the displacement over which individual forces act. But is easier to write the symbolic $F_{net}d\cos\theta$ than it is to write some sort of complicated summation over all individual forces, internal and external, acting over all their individual displacements. It represents the work done by all forces, internal and external, acting over their associated displacement, and neatly fits in with the standard representation of the Theorem which is for a single point mass object as the system. This then covers a single extended object, as on the previous page, as well as external forces that act on the individual masses that make up the system if the system consists of more than a single mass.

The energy of a system is composed of the kinetic energy, as well as potential energy or other internal energies. The potential energy and other internal energies are always the result of the interactions between the objects within the system and internal forces within the individual masses.

Therefore, the term W_{net} is the work done by all forces, internal and external, and includes the energy required for the change in the potential energy within the system if it is composed of more than one object as well as work involving non-conservative forces such as friction.

Example: Spring compressed x_0 and placed between two equal masses. Upon release, the two masses move off with speed v. Assume no friction. Consider the system to be the two masses.



In a problem like this, the initial and final configurations would often be labeled the initial and final systems, which makes it a bit confusing since the system is defined to be only the two masses. I have added the FBD for the system of 2 masses. [It is strange looking because the spring is not part of the system.] The visual of the FBD makes clear what forces are acting on the system – the two normal forces gravitational forces acting on the two masses vertically, and the spring forces acting on the two masses horizontally with appropriate directions. Given this FBD, we can write:

$$\Delta KE = W_{net}$$

$$2\left(\frac{1}{2}mv^2\right) = W_{net}$$

$$= 2\left(\frac{1}{2}\left(\frac{k}{2}\right)\left(\frac{x_0}{2}\right)^2\right)$$ since half the spring pushes each mass half the compressed length x_0

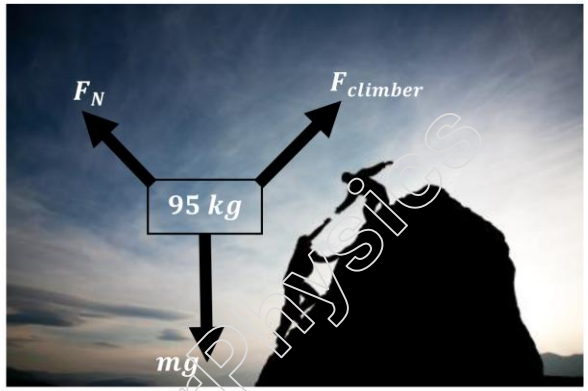
$$= \left(\frac{1}{2}kx_0^2\right)$$ which was the work required to compress the spring to create the initial system

This initial $\frac{1}{2}kx_0^2$ is the *internal energy* of the initial system which is available to do work on the masses, accelerating them to speed v in the final system.

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Work-Energy Theorem Example

Find the work done by a $95\ kg$ climber to climb a $1500\ m$ tall mountain.



$$W_{net} = \Delta KE$$

$$W_{mg} + W_N + W_{climber} = KE_f - KE_i$$

$$-mgh + 0 + W_{climber} = 0 - 0$$

The climber pushes down the slope against the mountain, the mountain pushes the climber forward. $F_{climber}$ is the force of the mountain pushing the climber up. This is constantly changing in both direction and magnitude! Impossible to calculate directly from $Fdcos(\theta)$.

F_N is always perpendicular to the direction of motion.

F_{mg} is constant.

So:

$$W_{climber} = mgh$$

$$W_{climber} = (95)(100)(1500)$$

$$W_{climber} = 1.42 \times 10^7\ J$$

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Work Energy Theorem Worksheet

1. Abercrombie ($75\ kg$) jumps out of a plane 1.0 km above the ground. Find the work done by gravity during his fall.

2. After opening his parachute, Abercrombie reaches a terminal velocity of 10 m/s. What is the work done by friction during his descent to the ground?

3. Belinda (70 kg) climbs a 15 m tall rock wall. How much work does she do to reach the top?

4. Calvin rides a 1500 kg roller coaster and is pulled to the top of a 50 m tall ramp at a constant speed. How much work does the cable do on the roller coaster, assuming no friction?

5. Draco the crane operator lifts a 5600 kg steel beam to the top of a 75 m tall building under construction. How much work did the crane do?

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Work Energy Theorem Worksheet Page 1

Student Practice Blank

Work Energy Theorem Worksheet

6. Eze (60 kg), only recently learning to skateboard competitively, starts at the top of a 4.0 m tall half-pipe. He only makes it half-way up the other side. How much work was done by friction?



Fiona jumps off the top of a 5.0 m tall diving diving-board. Her mass is 60 kg. What is her speed as she hits the water? Assume no air friction.

7. After entering the water, Fiona then sinks 2.0 m before stopping and rising to the surface. How much work did the water do in stopping her descent? (Careful – don't forget gravity!)

8. Grover pushes a 15 kg desk across the floor with a constant force of 25 N. The desk moves 5.0 m in 4.0 s. What is the power generated by Gareth during his effort?

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Work Energy Theorem Worksheet Page 2

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Conservation of Energy

Conservation of Energy

In developing the Work-Energy Theorem earlier, the principle of the conservation of energy was invoked, leading to $\Delta KE_{total} = W_{net}$. All of the complexity comes from the work done by the forces acting on, and within, the system.

A great simplification occurs when the only forces that act are conservative forces within the system. Conservative forces were defined earlier in the Forces unit. A force is conservative if the work done by that force is independent of the path taken in the transition from the initial to final state. In that case, one can define a potential energy, which is the negative of the work done. With this simplification $W_{net} = -\Delta PE_{total}$. Thus the Work-Energy Theorem transforms into

Conservation of Mechanical Energy

$$\sum KE_i + \sum PE_i = \sum KE_f + \sum PE_f$$

Mechanical energies are the kinetic and potential energies. The total mechanical energy of a system is conserved ***if and only if*** there are no non-conservative forces acting and no external forces acting.

Compare the above with

Conservation of Total Energy

$$E_{added} + \sum KE_i + \sum PE_i = \sum KE_f + \sum PE_f + E_{lost}$$

The Conservation of Total Energy, which is nothing more that a different way of writing the Work-Energy Theorem, ***is always valid***. E_{added} and E_{lost} are the energies added to, or removed from, the system due to external forces, or any non-conservative internal forces. The most common example is the energy lost due to friction acting on, or within, the system.

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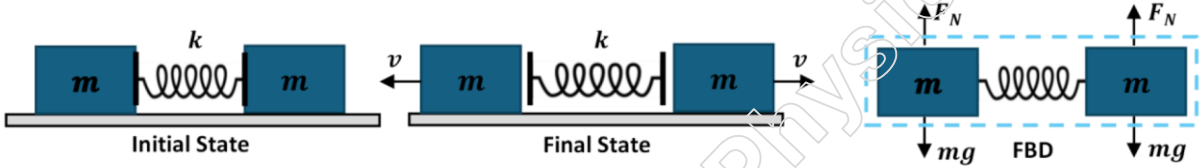
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Conservation of Energy

Conservation of Energy

The following example was analyzed earlier using the Work-Energy Theorem, where the system was defined as the two masses. Let's look at it through a conservation of energy lens where the system now includes the spring.

Example 1: Spring compressed by x_0 by two equal masses. Upon release, the two masses move off with speed v. The system consists of the two mass + spring. Assume no friction. Since there is no nonconservative forces acting, we can use Conservation of Mechanical Energy.



$$\sum KE_i + \sum PE_i = \sum KE_f + \sum PE_f$$

$$0 + \frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 0$$

In this case, the spring force is a conservative force, so potential energy was used. Compare this to the Work-Energy approach earlier, we had to calculate the work done by the spring in the Work-Energy case. [Of course, the potential energy is the negative of the work done by the spring, so choose whichever is easier.] The end result, of course, is the same.

Example 2: Now, let's assume that there is friction – suppose a coefficient of friction μ between the block and the table, and our final system occurs at the moment the spring has returned to its original uncompressed position. By symmetry, the two masses will have each moved a distance $\frac{x_0}{2}$, and the speed at this point is v', which is obviously less than v. Now, because of friction, we must use Conservation of Total Energy

$$E_{added} + \sum KE_i + \sum PE_i = \sum KE_f + \sum PE_f + E_{lost}$$

$$0 + \ 0 + \frac{1}{2}kx_0^2 = \frac{1}{2}m(v')^2 + \frac{1}{2}m(v')^2 + \mu mg\left(\frac{x_0}{2}\right) + \mu mg\left(\frac{x_0}{2}\right)$$

To come full circle, we can compare this result to the Work-Energy Theorem approach:

$$\Delta KE = W_{net} = W_{spring} + W_{friction}$$

$$\frac{1}{2}m(v')^2 + \frac{1}{2}m(v')^2 = 2\left(\frac{1}{2}\left(\frac{k}{2}\right)\left(\frac{x_0}{2}\right)^2\right) - 2\left(\mu mg\left(\frac{x_0}{2}\right)\right)$$

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General Procedure Energy Conservation

Conservation of Energy Procedure

- Identify the initial and final points
- Box and sum PE and KE equations at initial point
- Box and sum PE and KE equations at final point
- If Mechanical Energy is conserved, these two boxes are equal in value
  - $KE_i + PE_i = KE_f + PE_f$
- Otherwise, the difference between the boxes is, typically, E_{lost}
  - $E_{added} + KE_i + PE_i = KE_f + PE_f + E_{lost}$
  - It should be kept in mind that E_{lost} is conceptually a positive energy
  - If the question asks for the work done by friction, $W_f = -E_{lost}$

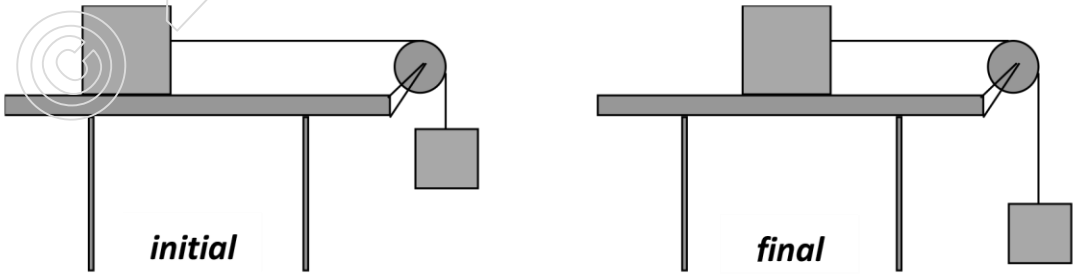
Concerns:

There may be more than one potential energy

- **perhaps a spring and gravity for example**

If there are more than one mass, then there can be more than one kinetic energy, and the "initial" and "final" points involves multiple physical points, making the sketch a bit more problematic

- **Below might be an appropriate initial and final "points" for a pendulum system**

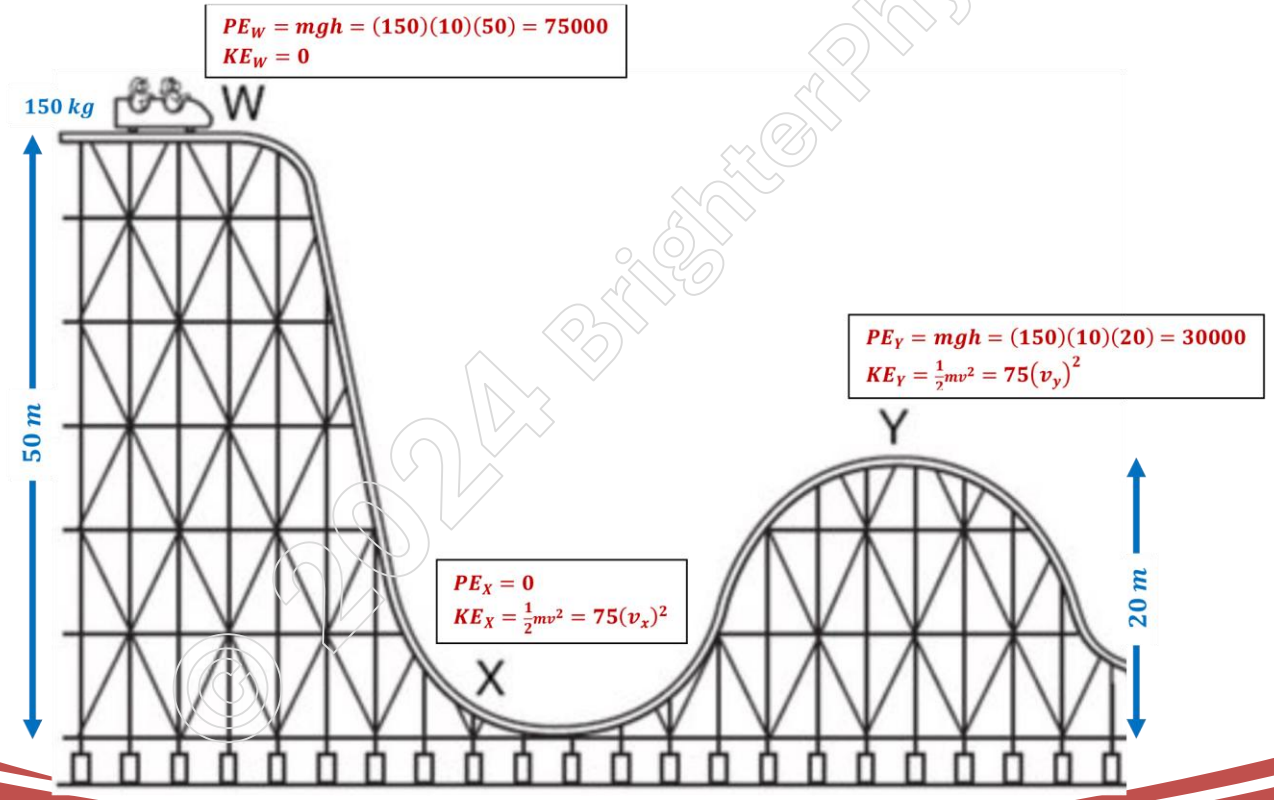


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Conservation of Energy Example

A $150\ kg$ roller coaster car starts from rest at Point W, $50\ m$ above point X, and rolls down the track through Point Y which is $20\ m$ above Point X.

The first step is to box the KE and PE at the initial and final points. We will be using all three points, so box all three.

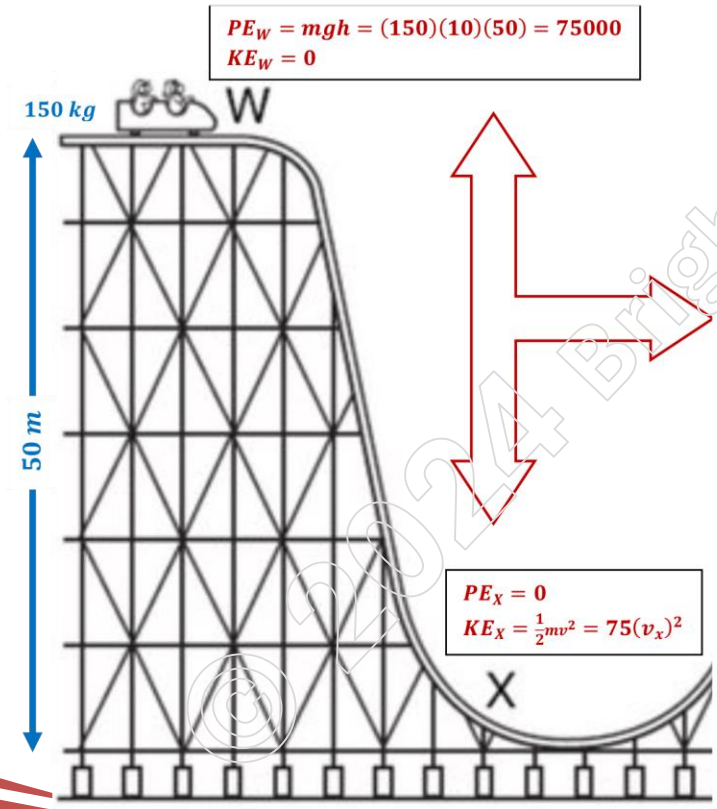


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Conservation of Energy Example

A $150\ kg$ roller coaster car starts from rest at Point W, $50\ m$ above point X, and rolls down the track through Point Y which is $20\ m$ above Point X.

Find the kinetic energy at Point X assuming that the track is frictionless.



$$PE_W + KE_W = PE_X + KE_X$$

$$75000 + 0 = 0 + KE_X$$

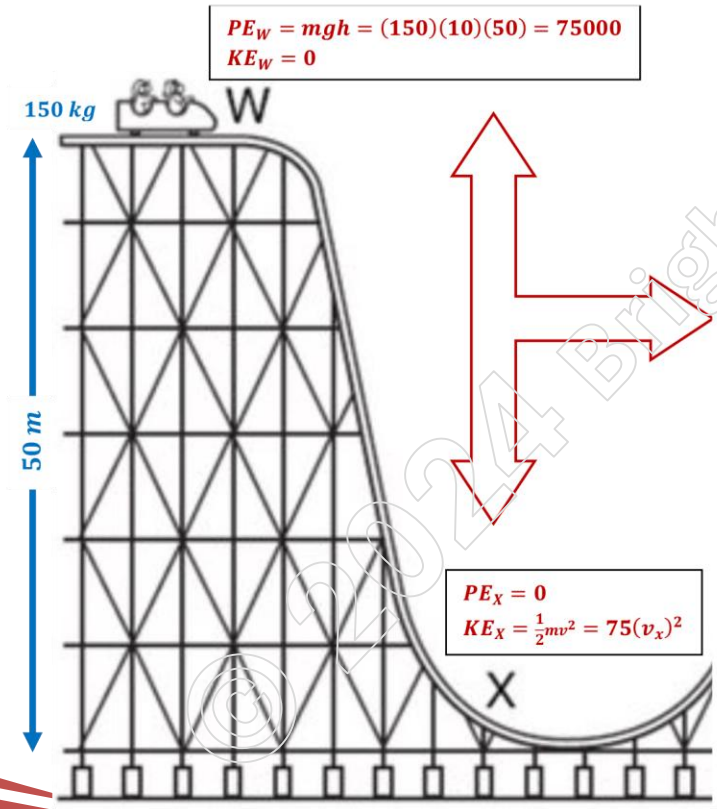
$$KE_X = 75000\ J$$

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Conservation of Energy Example

A $150\ kg$ roller coaster car starts from rest at Point W, $50\ m$ above point X, and rolls down the track through Point Y which is $20\ m$ above Point X.

Find the speed at Point X assuming that the track is frictionless.



$$PE_W + KE_W = PE_X + KE_X$$

$$75000 + 0 = 0 + 75(v_X)^2$$

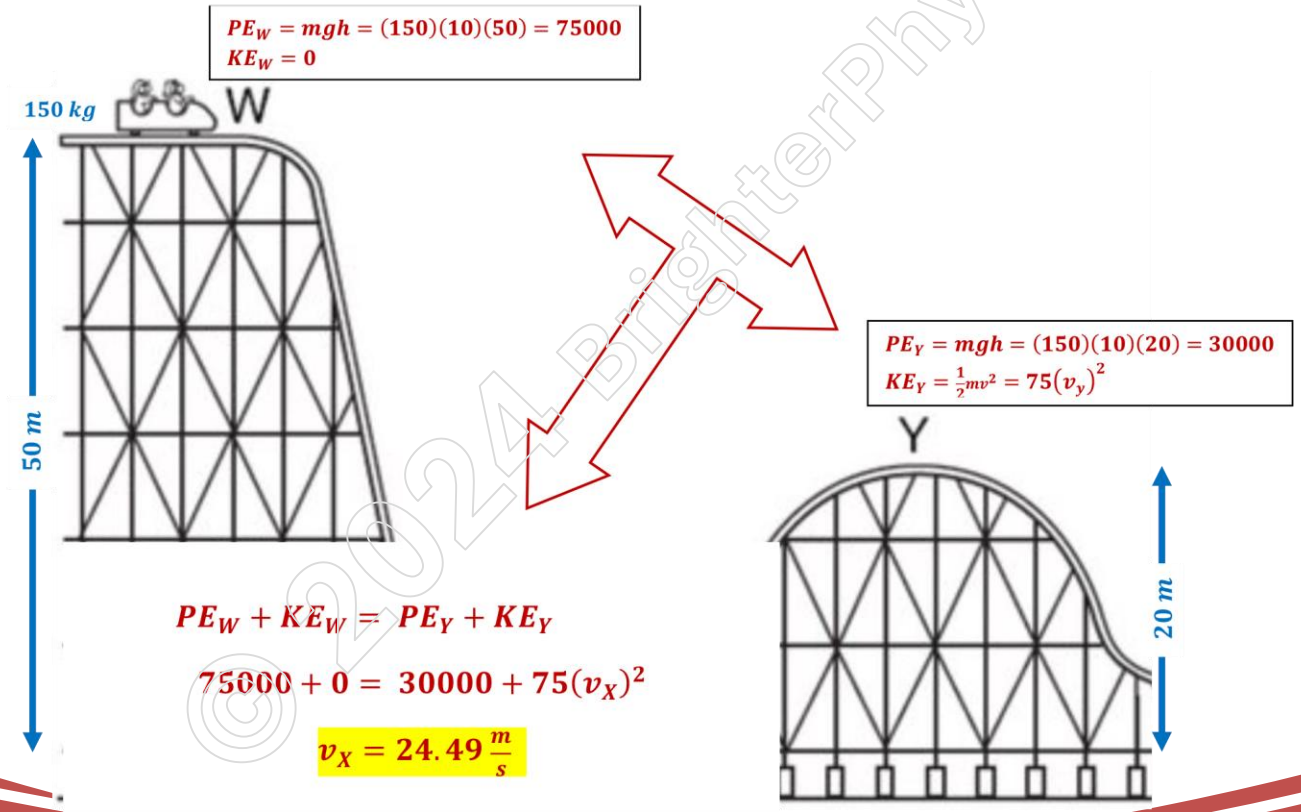
$$v_X = 31.62\ \frac{m}{2}$$

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Conservation of Energy Example

A $150\ kg$ roller coaster car starts from rest at Point W, $50\ m$ above point X, and rolls down the track through Point Y which is $20\ m$ above Point X.

Find the speed at Point Y assuming that the track is frictionless.

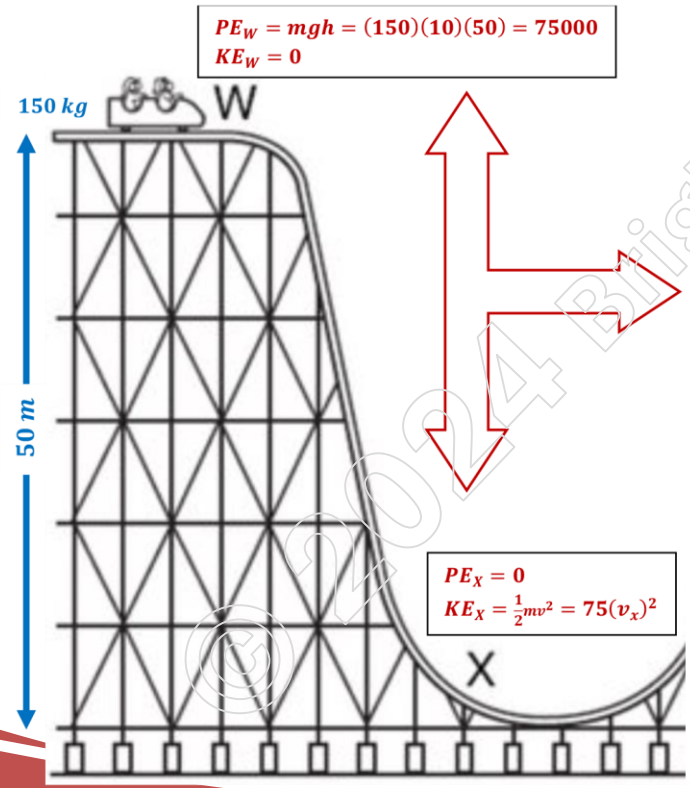


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Conservation of Energy Example

A $150\ kg$ roller coaster car starts from rest at Point W, $50\ m$ above point X, and rolls down the track through Point Y which is $20\ m$ above Point X.

Now suppose that the speed at point X is $20\ \frac{m}{s}$. Find the work done by friction during the motion from Point W to Point X.



Conservation of TOTAL Energy:

$$PE_W + KE_W = PE_X + KE_X + E_{lost}$$

$$75000 + 0 = 30000 + 75(20)^2 + E_{lost}$$

$$E_{lost} = 28125\ J$$

$$W_{friction} = -28125\ J$$

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Conservation of Energy Worksheet

1. A 100 N sack of grain is hoisted to a storage room 50 m above the ground floor of a grain elevator.

a) How much work was done to raise the sack of grain 50 m?

b) What is the increase in potential energy of the sack of grain at this height compared to at the ground?

c) The rope being used to lift the sack breaks just as the sack reaches the storage room. What kinetic energy does the sack have just before it strikes the ground?

2. A 20 kg rock is on the edge of a 100 m cliff.

a) What potential energy does the rock possess relative to the base of the cliff?

b) The rock falls from the cliff. What is its kinetic energy just before striking the ground?

c) What speed does the rock have as it strikes the ground?

3. An archer puts a 0.3 kg arrow across the bowstring. An average force of 201 N is exerted to draw the string back 1.3 m.

a) Assuming that all of the energy goes into the arrow, with what speed does the arrow leave the bow?

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Conservation of Energy Worksheet Page 1

Student Practice Blank

4. A 2.0 kg rock initially at rest loses 407 J of potential energy while falling to the ground.

a) Calculate the kinetic energy that the rock gains while falling.

b) What is the rock's speed just before it strikes the ground?

5. A Physics book of unknown mass is dropped 4.50 m. What speed does the book have just before it hits the ground?

6. Wilhelmina weighs 420 N, and she is sitting on a playground swing that hangs 0.40 m above the ground. Her mom pulls the swing back and releases it when the seat is 1.00 m above the ground.

a) How fast is Wilhelmina moving when the swing passes through its lowest point, assuming no friction?

b) If Wilhelmina moves through the lowest point at 2.0 $\frac{m}{s}$, how much work was done by friction to slow the swing?

7. Gunther throws a 10.0 g ball straight down from a height of 2.0 m. The ball strikes the floor with a speed of 7.5 $\frac{m}{s}$. What was the initial speed of the ball?

8. Bertha's mass is 28 kg. She climbs the 4.8 m ladder of a slide and reaches a speed of 3.2 $\frac{m}{s}$ at the bottom of the slide. How much work was done by friction on Bertha?

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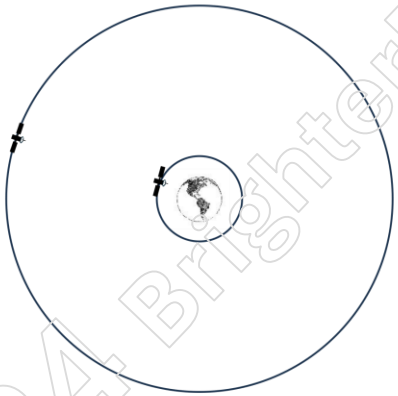
Conservation of Energy Worksheet Page 2

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Conservation of Energy - InterOrbital Worksheet

A 1000 kg satellite is launched into the same orbit as the International Space Station (ISS). What is the energy needed to raise the satellite into a Geosynchronous orbit (GEO) above the Earth?

$R_{GEO} = 4.22 \times 10^7\ m$ $R_{ISS} = 6.81 \times 10^5\ m$



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Conservation of Energy - InterOrbital Worksheet

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AP Consistency Issues & Energy

AP Consistency Issues & Energy

Throughout these units I have paid close attention to defining the system as part of the problem solving process, incorporating that choice explicitly in the procedures described. So, for example, if a problem involves objects A and B, then three separate systems are available to analyze: A by itself, B by itself, and the combination A&B. And I have been careful to identify, when appropriate, forces external, and internal to the system.

When it comes to energy problems, the AP places too much emphasis on system definitions than is really warranted. The choice of the system is properly made by the person solving the problem. Whether we say a ball gains speed as it rolls down a hill because gravity is doing work on the ball or gravitational potential energy is converted into kinetic energy gets us to the same correct answer.

That said, however, the AP writes the questions for AP tests, so….

The problem is that the definitions the AP uses regarding systems is not consistent with most sources. As far as being able to solve problems, it really does not matter. But the AP does tend to ask definitional questions about systems in terms of energy problems. Typically these are in the multiple choice part of the test, but occasionally shown up as a small part of a free-response question.

The problem has to do with the words closed, open, and isolated systems. In short, the AP equates closed and isolated systems as systems that have no net external forces acting on them. Whereas an open system does have a net external force acting.

This is at odds with the majority in the Physics world where closed means no mass enters or leaves the system, whereas for an open system the mass is not conserved. And isolated means that there is no net force acting on the system.

On the next few slides, a few examples of this type of questioning are presented.

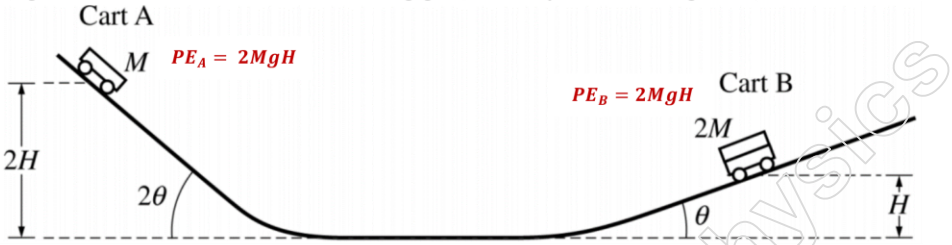
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AP Consistency Issues & Work

Example 1

Cart A has mass M and is released from rest at a height $2H$ on a ramp making an angle 2θ with the horizontal. Cart B has mass $2M$ and is released from rest at a height H on a ramp making an angle θ with the horizontal. The carts roll toward each other, have a head-on collision on the horizontal portion of the ramp, and stick together. The masses of the carts' wheels are negligible, as are any frictional or drag forces.



a) Indicate, and explain, whether the carts remain at rest, move to the left, or move to the right after the collision.

For each cart, ME is conserved before the collision:

$PE_{top} + 0 = 0 + KE_{bottom} = \frac{p^2}{2m}$ So $p = \sqrt{2mPE}$

$\{p_A = 2M\sqrt{gH}\}$ < $\{2M\sqrt{2gH} = p_B\}$ So, after collision, the move off to the left because momentum is conserved

b) For the system consisting of the two carts and Earth, indicate whether the total mechanical energy increases, decreases, or remains the same.

For each cart, ME is conserved before the collision, and is therefore a constant up until the collision occurs.

The collision is a non-conservative internal force that is removing mechanical energy (creating heat, noise, deformations, …)

So the total mechanical energy of the system decreases.

c) For the system consisting of only the two carts, indicate whether the total mechanical energy increases, decreases, or remains the same.

For each cart, $ME = 0$ initially. There is no potential energy since Earth is not part of the system and initially neither cart is moving.

The gravitational force does positive work on each cart increasing its mechanical energy.

After the collision, the carts are both moving to the left – they have kinetic energy. They have positive mechanical energy.

The carts begin with $ME = 0$, and end with $ME > 0$.

So the total mechanical energy of the system increases.

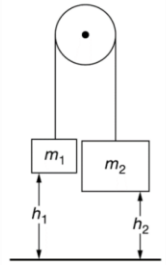
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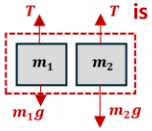
AP Consistency Issues & Work

Example 2

The Atwood's machine shown consists of two blocks of mass m_1 and m_2 that are connected by a light string that passes over a pulley of negligible friction and negligible mass. The block of mass m_1 is a distance h_1 above the ground, and the block of mass m_2 is a distance h_2 above the ground. m_2 is larger than m_1. The two-block system is released from rest. Which of the following claims correctly describes the outcome after the blocks are released from rest but before the block of m_2 reaches the ground? Select two answers.



a) For the system consisting of the two blocks, the change in the kinetic energy of the system is equal to the work done by gravity on the system.



$$\Delta KE = W_{net}$$
$$\Delta KE = (m_2 - m_1)gh_2 + (T - T)h_2$$
$$\Delta KE = (m_2 - m_1)gh_2$$

This is a **correct** restatement of the Work Energy Theorem applied to the two mass system

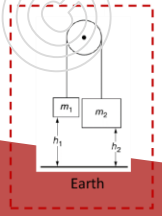
b) For the system consisting of the two blocks, the change in kinetic energy of the system is equal to the sum of $m_1gh_1 + m_2gh_2$.

Incorrect: m_2gh_2 is the work done by gravity on m_2 during the motion. m_1gh_1 is nothing – it would be the work on m_1, but it does not move a distance h_1. These would be the initial potential energies if the Earth was part of the system, but it is not.

c) For the system consisting of the two blocks and the pulley, the increase in kinetic energy of the system is equal to the decrease in the gravitational potential energy of the system.

Incorrect: There is no potential energy in this system!

d) For the system consisting of the two blocks, the pulley, and Earth, the change in the total mechanical energy of the system is zero.



$$\Delta KE = W_{net}$$
$$\Delta KE = 0$$
$$No\ external\ forces; no\ friction$$

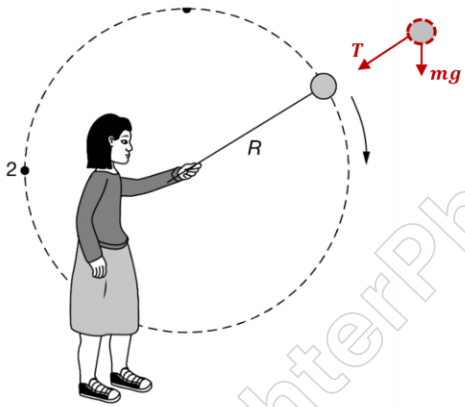
This is a **correct** restatement of the Conservation of Energy applied to the pulley-mass-Earth system

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AP Consistency Issues & Work

Example 3

A student swings a ball at the end of a string of length R in a vertical circle as shown. The ball moves with the minimum speed necessary to complete a circular path at point 1. Consider the system of the ball. In terms of the mechanical energy of the system, which of the following claims is correct, and why?



a) The system is open, because the ball is able to complete a full revolution around the circle.

Since the ball is moving in circle, there must be a net force toward the center (although it might be zero at the bottom point). But this is an implication, not a direct statement.

b) The system is open, because a net force is exerted on the ball as it travels in a circle.

The **correct** definition of an open system – there exists an external force on the system (which in this case, is the ball).

c) The system is closed, because the ball is able to complete a full revolution around the circle.

There is a net force – gravity and rope – acting on the system. The system is not closed.

d) The system is closed, because the speed of the ball remains constant as it travels in the vertical circle.

There is a net force – gravity and rope – acting on the system. The system is not closed.

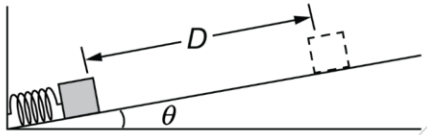
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AP Consistency Issues & Work

Example 4

A block of mass M is at rest on a ramp that is inclined at an angle θ with respect to the horizontal. Frictional forces are considered to be nonnegligible. The block is pushed against a spring and then held in place. The spring is compressed a distance of x_1, and the spring is not secured to the block. The block is then released from rest, travels up the incline, and comes to rest after traveling a distance D, as shown above. Which of the following claims correctly describes the energy of the system under consideration from when the block compressed the spring and when the block has traveled a distance D along the incline? Select two answers.



a) The mechanical energy of the system consisting of the spring increases by $\frac{1}{2}k(x_1)^2$.

The potential energy of the spring decreases by $\frac{1}{2}k(x_1)^2$. It has no mass and so has no kinetic energy. So Mechanical energy decreases.

b) **The mechanical energy of the system consisting of the block does not change.**

Initially the block is not moving. Finally the block is not moving. The block, as the system, has no potential energy. So $\Delta ME = 0$.

c) **The mechanical energy of the system consisting of the block and Earth increases by more than zero but less than $\frac{1}{2}k(x_1)^2$.**

As the spring expands, it does $\frac{1}{2}k(x_1)^2$ work on the block-Earth system increasing the mechanical energy. However, friction is decreasing the mechanical energy as the block continues to rise.

d) The mechanical energy of the system consisting of the spring, block, and Earth increases by $\frac{1}{2}k(x_1)^2$.

The initial ME of the spring-block-Earth system is $\frac{1}{2}k(x_1)^2$ (if the lowest point of the block has zero gravitational potential energy). As the system evolves, some ME is lost to friction decreasing the ME overall.

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