Torque

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Forces I

Torque
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Introduction to Torque

Introduction to Torque

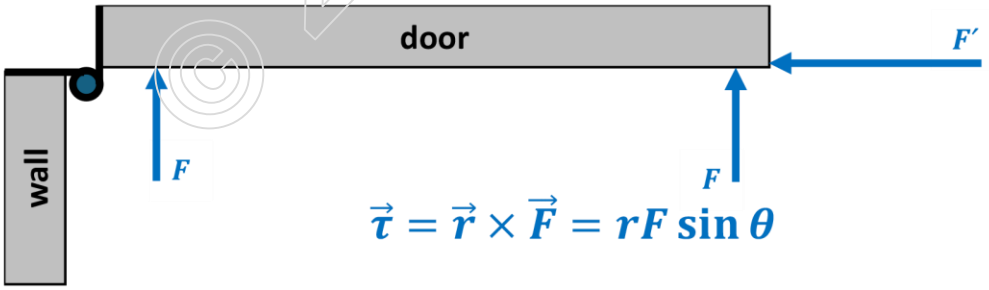
As with kinematics, rotational force or torque, can be taught quickly by using the linear analogue.

- **The (net) force acting on a system causes its state of motion, its velocity, to change. The system accelerates linearly.**
- **The (net) torque acting on a system causes it state of angular motion, its angular velocity, to change. The system accelerates angularly.**

I find the following class demo very useful for the students. It is simple, accessible, identifies the quantities involved in, and demonstrates the vector nature of, torques. A door is attached to an adjacent wall by, and rotates around, a hinge.

If the same force F is applied to two different positions as shown, one close to the hinge and the other far from the hinge, the resulting motion of the door is markedly different. I simply use my finger, not my arm, to push. The force close to the hinge barely moves the door, but the force farther from the hinge results in a large angular motion of the door. And if the force is increased, the resulting angular motion increases, easily demonstrated. This identifies that the torque is dependent on both the force applied and the distance from the axis of rotation at which it is applied.

A second force F' applied directly toward the hinge will result in no angular motion, regardless of how large a force. It can be made very dramatic by leaning into the door to create a large force – but beware, even a small deviation in the angle will result in a very large and sudden rotational result for the door! This demonstrates that the angle at which the force is applied with respect to the direction toward the axis of rotation affects the resulting rotational motion of the door, demonstrating the vector nature of torque.



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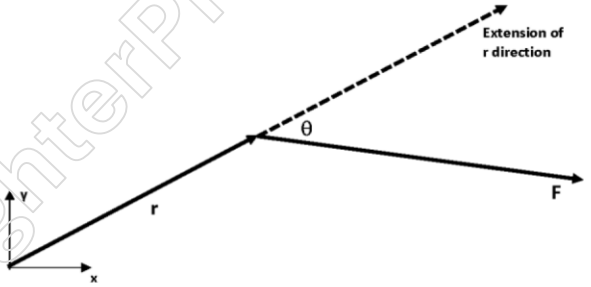
Introduction to Torque

As with kinematics, rotational force or torque, can be taught quickly by using the linear analogue.

Linear Motion	Rotational Motion
Force $(\vec{F})$	Torque $(\vec{\tau})$
Causes objects to change state of motion $\vec{v}$	Causes objects to change state of rotational motion $\vec{\omega}$
$\vec{F}_{net} \equiv m\vec{a}$	$\vec{\tau}_{net} = I\vec{\alpha}$
-	$\vec{\tau} \equiv \vec{r} \times \vec{F}$

Let's look at a general torque problem.

As with all rotational problems, all calculations depend on the choice of coordinate system. Generally, this is at the discretion of the person solving the problem. Choose wisely! Here, the x-y axes are shown at the bottom-left.



The vector $\vec{r}$ is the position of the object with respect to the coordinate system. The vector $\vec{F}$ is the force acting on the object at $\vec{r}$. The angle θ is the (smallest) angle between the direction of $\vec{r}$ and $\vec{F}$. Typically, as is the case here, all this is in the plane of the paper/screen.

The direction of the torque will be in the direction perpendicular to the plane formed by the vectors $\vec{r}$ and $\vec{F}$. The direction can be determined from the right-hand-rule, which in this case is into the plane in the negative z direction, or, as expected by the AP, by looking at how the force will tend to change the motion of the object w/t the coordinate system – in this case clockwise about the z-axis.

The magnitude of the torque is given by $rF\sin\theta = r_{\perp}F = rF_{\perp}$ where $r_{\perp}$ is the component of $\vec{r}$ in perpendicular to direction of $\vec{F}$, and $F_{\perp}$ is the component of $\vec{F}$ perpendicular to the direction of $\vec{r}$.

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What is Rotational Motion?

What is Rotational Motion?

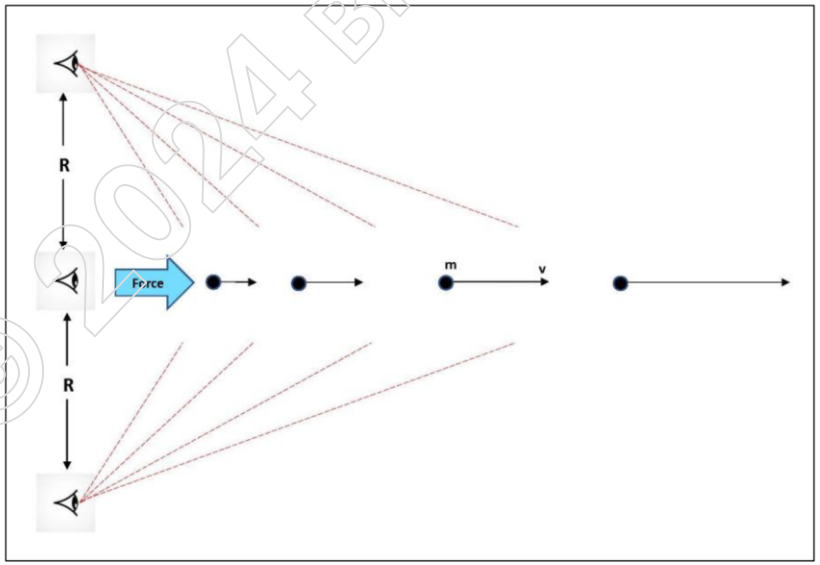
What does it mean for an object to be rotating? This is a different question than asking if an object is spinning. A top spins on its axis, the Earth spins around it axis. Yes, these are also rotations, but a rotation has a more general meaning.

Rotations can be confusing. The Earth rotates around the sun. A mass whirls around out hand as we swing it. We then, incorrectly, assume that an object must follow a curved path if it is to rotate about something.

In the following drawing, a mass $\boldsymbol{m}$ is moving horizontally with a velocity $\vec{v}$. A constant horizontal force $\vec{F}$ is acting on the mass as it moves.

Three different observers, separated by a distance R from each other are shown at left:

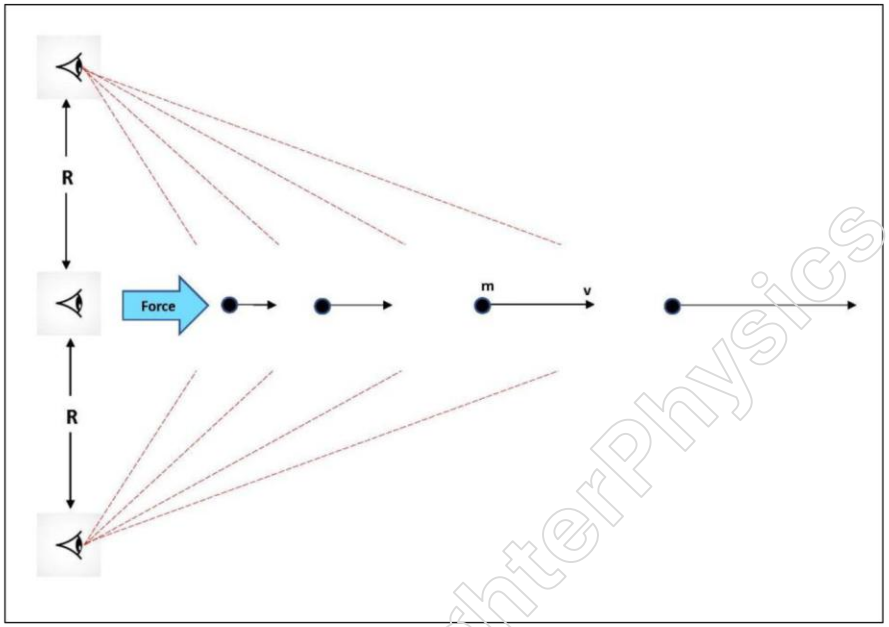
- The center observer says that, as time goes on, she does not have to change her line of sight. The mass is not rotating.
- The top observer says that, as time goes on, he has to rotate his line of sight counterclockwise to continue to see the mass. The mass is rotating counterclockwise about him.
- The bottom observer says that, as time goes on, they have to rotate their line of sight clockwise to continue to see the mass. The mass is rotating clockwise about them.



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What is Rotational Motion?



The mass is moving in a straight line to the right the entire time.

However, each observer, using different coordinate systems (in this case, their own locations) to view the same event, observes distinctly different angular motions. [They each also measure different values for the radial coordinate of $\vec{r}$, by the way. But that has nothing to do with "rotation".]

Looking at the torques measured by the three different observers:

- The center observer says that the torque acting on the mass is 0 because $\vec{r}$ and $\vec{F}$ are parallel.
- The top observer says that the torque acting on the mass is counterclockwise with a magnitude of FR.
- The bottom observer says that the torque acting on the mass is clockwise with a magnitude of FR.

The value of a torque MUST reference the coordinate system

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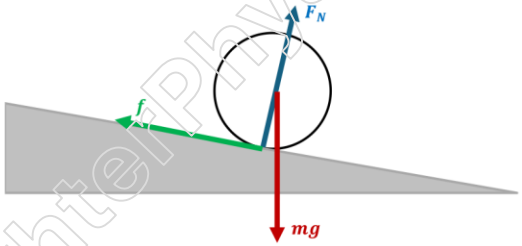
Problems Involving Torque

Problems Involving Torques

Given a problem that involves torques, we will necessarily have to consider forces also since there is no torque without a force acting. So the procedures that we developed for solving force problems complement what needs to be done for torques.

In particular, an FBD is very helpful identifying the system under consideration and the external forces acting. In a pure force problem, we only need to know what is inside our system boundary, and are not concerned about the arrangement within the FBD, nor where the external forces are acting within the system. By carefully showing where the external force is acting within the system then serves the needs for both the force and the torque parts of the problem.

For example, the diagram at right shows a ball rolling down a ramp. The system is the ball. The gravitational force $\boldsymbol{mg}$ acts at the center of mass of the ball, the normal force $\boldsymbol{F_N}$ and friction $\boldsymbol{f}$ both act at the point of contact of the ball and the ramp.



Given this expanded FBD, we could now apply Newton's 2nd Law, $\vec{\boldsymbol{F}}_{\boldsymbol{net}} = \boldsymbol{m}\vec{\boldsymbol{a}}$, as we have before, and now also Newton's Rotational 2nd Law, $\vec{\boldsymbol{\tau}}_{\boldsymbol{net}} = \boldsymbol{I}\vec{\boldsymbol{\alpha}}$. There are three forces acting, so there will be three torques that will need to be determined for this rotational equation.

To determine the torques, we need to choose a point about which to calculate the torques, as well as the moment of inertial $\boldsymbol{I}$. There are an infinite number of points to choose from. There are only two good ones: the point of contact and the **center of mass**, this last being almost always the best choice, if for no other reason than we will not have to use the Parallel Axis Theorem since $\boldsymbol{I}_{\boldsymbol{cm}}$ is usually given and, frankly, most would forget, or not recognize, that the moment of inertia also depends on the same choice of rotational axis.

Finally, there is often a need to connect the rotational to the linear. Here the linear acceleration of the center of mass and the angular acceleration are related if the ball rolls without slipping, which is almost always the case for AP1. [This also mean that the friction is static, not kinetic, friction. So $\boldsymbol{f} \leq \boldsymbol{\mu F_N}$.] To do this the sense of positive needs to be consistent. For the problem above, the acceleration is down the ramp and the angular acceleration is clockwise which is then the best choice for positive.

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Torque Worksheet

1. A man having a mass of 100 kg walks to the end of a diving board that is 2 m long. What torque does his weight produce about the fixed end?

2. Find the torque produced by a 300 N force applied to the end of a 1.5 m wide door at an angle of 90°.

3. Find the torque produced by a 300 N force applied to the end of a 1.5 m wide door at an angle of 0°.

4. Find the torque produced by a 300 N force applied to the end of a 1.5 m wide door at an angle of 60°.

5. What torque is produced by a 0.25 kg 90 mph baseball which is caught by a 2 m tall first-baseman? Assume that the ball comes to rest in the glove in 0.01 sec

6. A 40 kg boy sits 2 m to the left of the pivot of a long seesaw. A 30 kg girl sits 6 m to the right. Where must a 50 kg child be placed to just balance the seesaw?

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Torque Worksheet Page 1

Student Practice Blank

7. A window washer is standing on a scaffold supported by vertical ropes at each end. The scaffold weighs 200 N and is 3 m long. What is the tension in each rope when the 700 N worker stands 1 m from one end?

8. A massless meterstick with six suspended masses is placed on a fulcrum at the 50 cm mark. If a 10 g mass is suspended at the 10 cm mark, a 20 cm mass is suspended at the 20 cm mark, a 40 cm mass is suspended at the 40 cm mark, a 60 g mass is suspended at the 60 cm mark, and a 90 g mass is suspended at the 90 cm mark. At what position on the meterstick do you suspend a 56 g mass so that the meterstick is balanced? [Why does it not matter that the meterstick is massless?]

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Torque Worksheet Page 2

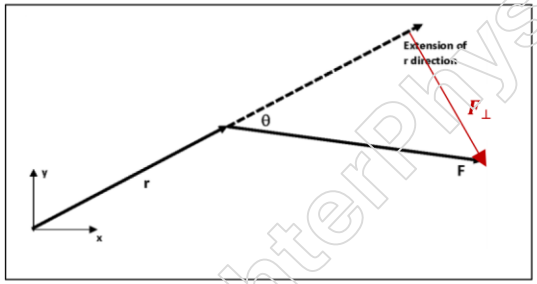
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Moment of Inertia

Moment of Inertia – Point Mass

Quick Derivation:

Suppose there is a point mass $\boldsymbol{m}$ at position $\vec{\boldsymbol{r}}$ from our choice of an axis of rotation and some force $\vec{\boldsymbol{F}}$ acts on it. As we have seen, the torque acting on the mass is $\boldsymbol{\tau = rF_{\perp}}$, which we set equal to $\boldsymbol{I\alpha}$.



But $\boldsymbol{F_{\perp} = ma_{\perp}}$, so $\boldsymbol{rma_{\perp} = I\alpha}$. As we have already seen in the rotational kinematics unit, $\boldsymbol{a_t = r\alpha}$, but here $\boldsymbol{a_t = a_{\perp}}$. So we can write $\boldsymbol{rma_{\perp} = rm(r\alpha) = I\alpha}$. Therefore, for a point mass, $\boldsymbol{I = mr^2}$.

Now, if the mass moves around in space only under the influence of the applied force, then the moment of inertia $\boldsymbol{I}$, as well as $\boldsymbol{\alpha}$, is dependent on $\boldsymbol{r}$. This is why we write $\boldsymbol{I\alpha}$ for torque only when the mass is at a fixed distance $\boldsymbol{r}$ from the axis of rotation.

Like mass, the moment of inertia is properly a scalar quantity. If we have a system of point masses, the total moment of inertia is simple the algebraic sum of the individual moments of inertia:

$$\boldsymbol{I_{sys} = \sum m_i r_i^2}$$ for a system of point masses.

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Moment of Inertia Video

Moment of Inertia

For extended objects:

$$I_{sys} = \int r^2 \, dm.$$

Students will not have to derive the moment of inertia using this formula. The moments of inertia about the centers of mass for whatever objects are needed will be given.

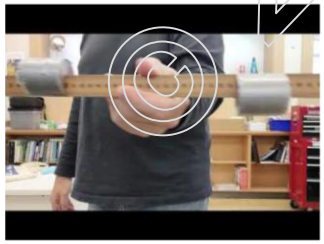
By far for Physics 1 students, the most common extended object moments of inertia about the center of mass are:

Hollow Hoop of mass M and radius R: $I_{hoop} = MR^2$

Solid Uniform Disk of mass M and radius R: $I_{disk} = \frac{1}{2}MR^2$

Moment of Inertia Demo:

https://youtu.be/OhuI9idzPpM?si=VEmjCnBTaQntPaBl



In this video:

- **the placement of the masses affects the moment of inertia**
- **the choice of rotation axis affects the moment of inertia**

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Moment of Inertia

Students will only need to be able to know:

- moment of inertia of point masses: $(\boldsymbol{mr^2})$
- moment of inertia of hollow hoop about the center of mass: $(\boldsymbol{MR^2})$
- moments for all other shapes around the center of mass will be given

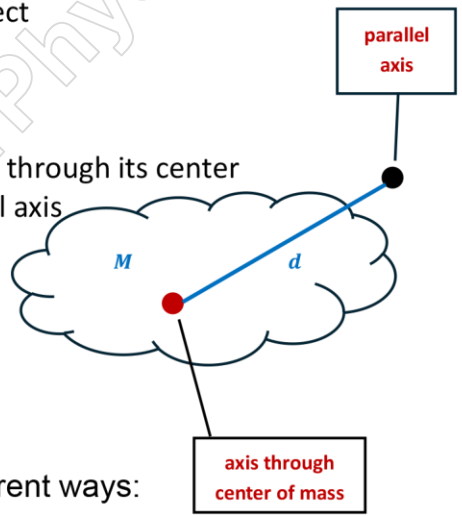
Qualitative statements:

- $\boldsymbol{I_{hollow} > I_{solid}}$ for round/spherical objects of mass M and radius R
- $\boldsymbol{I_{cm}}$ is the smallest moment of inertia for a given object

Parallel Axis Theorem:

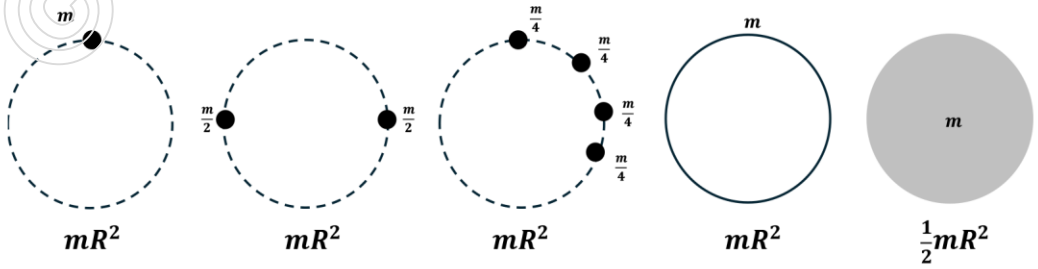
If the moment of inertia for a mass $\boldsymbol{M}$ around an axis through its center of mass, $\boldsymbol{I_{cm}}$, the moment of inertia around a parallel axis a distance $\boldsymbol{d}$ from the center of mass is given by:

$$\boldsymbol{I_d = I_{cm} + Md^2}$$



Each object below has a total mass m arranged in different ways:

- Each circle has the same radius R
- The dotted circles are massless hoops to which the point masses are attached
- The last is a uniform solid disk of mass m
- The moment of inertial about the center of the circle is below each figure



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Rotational-Torque Four Square

For each problem below, consider a 200 kg, 4.0 m long see-saw. The see-saw has position markings, like a ruler.
Moment of Inertia of a rod of mass M and length L about the center-of-mass is $\frac{1}{12}ML^2$

1. The see-saw pivots on a fulcrum positioned at 2.0 m, the center-of-mass of the see-saw. Your 65 kg friend sits on one end, at 0.0 m. Where should your 100 kg enemy sit in order to balance the see-saw?	**2.** The see-saw pivots on a fulcrum positioned at 1.75 m. Your 65 kg friend sits on one end, at 0.0 m. Where should your 100 kg enemy sit in order to balance the see-saw?
3. What is the Normal Force exerted by the fulcrum on the see-saw in problem 1.	**4.** What would be the angular acceleration of the see-saw in problem number 1 if you enemy suddenly got off the see-saw.

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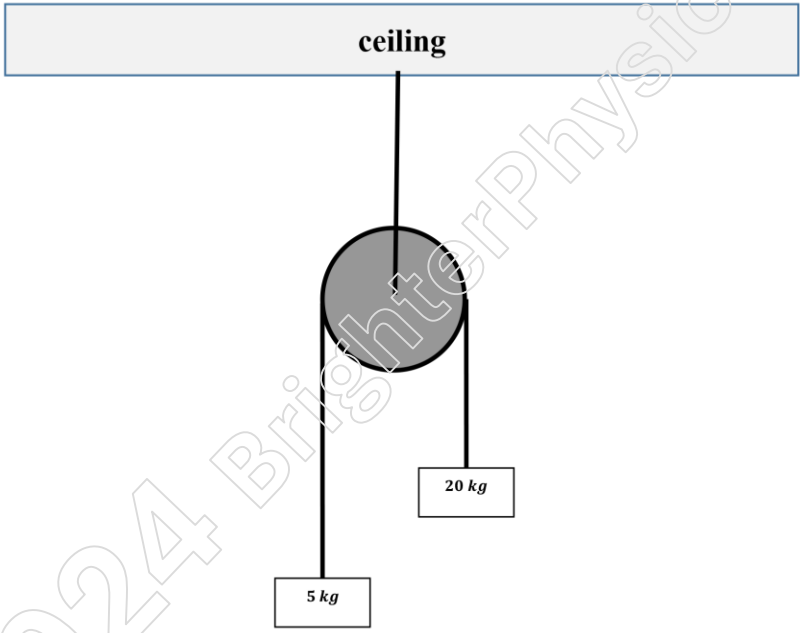
Rotational Torque Four Square

Student Practice Blank

Massive Pulley Quiz

A $M = 5.0\ kg$ frictionless pulley is suspended from a ceiling. Two masses, one $5\ kg$, and the other $20\ kg$ are connected by a massless rope that passes over the pulley. Assume that the pulley is a solid uniform disk, and thus $I_{pulley} = \frac{1}{2}MR^2$.

a) Find the tensions in the rope
b) Find the acceleration of the two masses.



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Ramp Race
-
Effects of Moment of Inertia Video

Ramp Race
Effect of Moment of Ineria Video

The link to a video that I made during the pandemic is offered below:

Ramp Race – The Effect of Moment of Inertia:

https://youtu.be/rsm1GqjAGkg?si=CI9ObZcOH7w22hs_



A demonstration of the effect of the moment of inertia on the acceleration of rolling object down a ramp.

The acceleration of objects of different geometries are directly compared. Solid vs hollow cylinders are used.

- **The radius does not matter – it cancels out.**
- **The mass does not matter – it cancels out.**

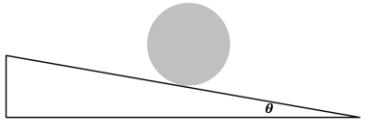
A very similar argument can be made using energy rather than acceleration instead.

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Rolling Mass on Ramp Worksheet

A uniform sold disk of mass $M = 5\ kg$ and radius $R = .3\ m$ rolls without slipping up a ramp that makes an angle of $\theta = 37°$ from the horizontal. What is the acceleration of the disk?



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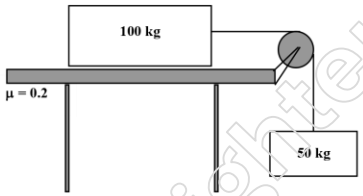
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Rolling Mass
on
Ramp
Worksheet

Student Practice
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Massive Pulley Worksheet

A 100 kg mass is placed on a table having a coefficient of friction $\mu = 0.2$. This mass is attached by light rope passing over a frictionless pulley to a hanging 50 kg mass. The pulley has a mass $M = 10\ kg$, radius $R = 0.25\ m$, and can be considered a uniform disk, so that its moment of inertial is $\frac{1}{2}MR^2$.
Find the tensions in the rope and the acceleration of the two masses.



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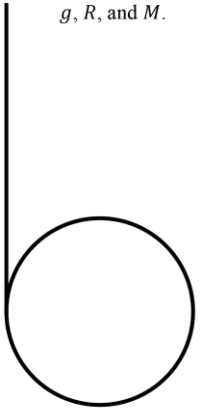
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Massive Pulley Worksheet

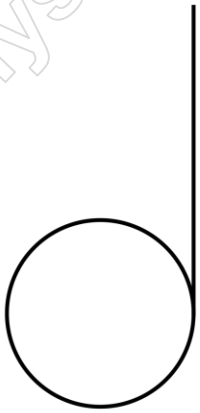
Student Practice Blank

Dual Falling/Rolling Disk Worksheet

A lightweight string is rolled around the edge of a **hollow ring** of mass M and radius R. Holding the end of the string, the ring is allowed to fall, unrolling the string as it goes. Find the acceleration of the ring in terms of g, R, and M.



A lightweight string is rolled around the edge of a **solid uniform disk** of mass M and radius R. Holding the end of the string, the disk is allowed to fall, unrolling the string as it goes. Find the acceleration of the disk in terms of g, R, and M.



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Dual Falling/Rolling Disk Worksheet

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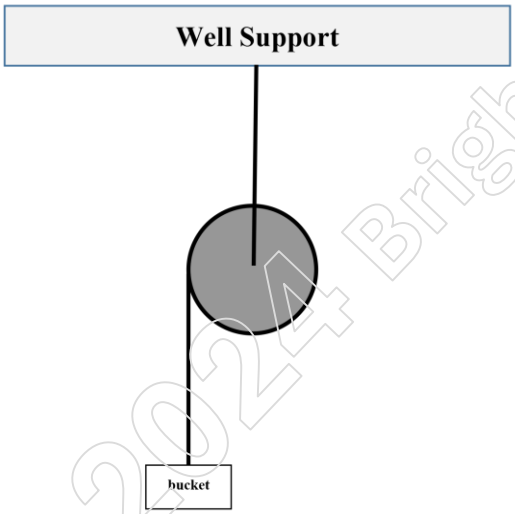
Water Well Worksheet

A 1.5 kg bucket is attached to a massless rope which wraps around the pulley attached to the water well support. Assume the pulley is a **uniform solid cylinder** and has a mass of 4.0 kg and a radius of 0.33 m. Suppose that there is a constant frictional torque of 1.10 $N \cdot m$ that acts on the pulley as it rotates.

The bucket is release from rest at the top of the well and is allowed to fall, pulling the rope which spins the pulley, as in the figure at right.



Find the acceleration of the bucket.



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Water
Well
Worksheet

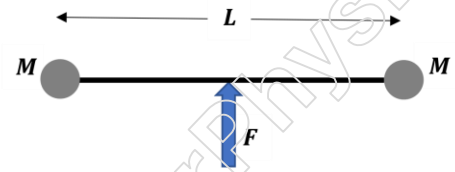
Student Practice
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Two Connected Masses Worksheet

1. Two identical mass M objects are connected by a massless rod of length L. A force F is applied to the mid-point as shown in the figure.

a) Find the acceleration of the center of mass, $\boldsymbol{a_{cm}}$ at the instant shown.

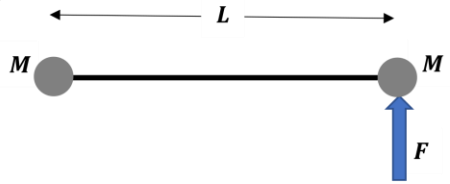
b) Find the angular acceleration, $\boldsymbol{\alpha}$, about the left mass at the instant shown.



2. Two identical mass M objects are connected by a massless rod of length L. A force F is applied to the right mass as shown in the figure.

a) Find the acceleration of the center of mass, $\boldsymbol{a_{cm}}$ at the instant shown.

b) Find the angular acceleration, $\boldsymbol{\alpha}$, about the left mass at the instant shown.



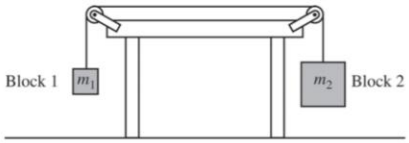
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Two Connected Masses Worksheet

Student Practice Blank

Torque AP Problem 2015



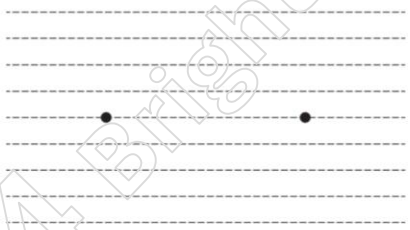
Note: Figure not drawn to scale.

1. (7 points, suggested time 13 minutes)

Two blocks are connected by a string of negligible mass that passes over massless pulleys that turn with negligible friction, as shown in the figure above. The mass m_2 of block 2 is greater than the mass m_1 of block 1. The blocks are released from rest.

(a) The dots below represent the two blocks. Draw free-body diagrams showing and labeling the forces (not components) exerted on each block. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.

Block 1 Block 2



(b) Derive the magnitude of the acceleration of block 2. Express your answer in terms of m_1, m_2, and g.

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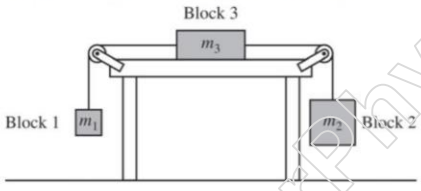
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Torque
AP Problem
2015
Worksheet
Page 1

Student Practice
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Torque AP Problem 2015

Block 3 of mass m_3 is added to the system, as shown below. There is no friction between block 3 and the table.



Note: Figure not drawn to scale.

(c) Indicate whether the magnitude of the acceleration of block 2 is now larger, smaller, or the same as in the original two-block system. Explain how you arrived at your answer.

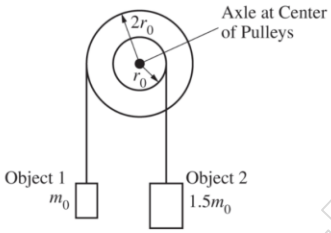
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Torque
AP Problem
2015
Worksheet
Page 2

Student Practice
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Torque AP Problem 2021



(7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.

m_0 is the mass of object 1.
$1.5m_0$ is the mass of object 2.
r_0 is the radius of the smaller pulley.
$2r_0$ is the radius of the larger pulley.

(a) At time $t = 0$, the pulleys are released from rest and the objects begin to accelerate.

i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of m_0, r_0, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

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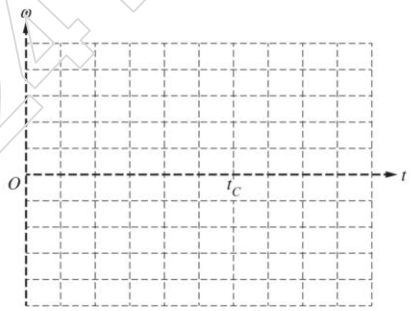
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(b) At a later time $t = t_C$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

___ Same direction ___ Opposite directions

Briefly explain your reasoning.

(c) On the axes below, sketch a graph of the angular velocity ω of the system consisting of the two pulleys as a function of time t. Include the entire time interval shown. The pulleys are released at $t = 0$, and the string is cut at $t = t_C$.



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