Rotational Kinematics

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The Rotational Kinematics Equations

The Rotational Kinematic Equations

For an object with constant angular acceleration:

I. $\omega_f = \omega_i + \alpha t$

II. $\omega_f^2 = \omega_i^2 + 2\alpha\theta$

III. $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

IV. $\theta = \frac{1}{2}(\omega_f + \omega_i)t$

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α = constant angular acceleration during the time interval

t = time interval for the motion

θ = angular displacement of object during time interval t

ω_i = initial angular velocity during the time interval

ω_f = final angular velocity during the time interval

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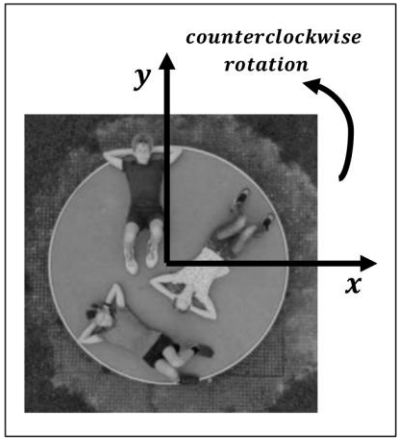
Introduction to Rotational Motion

Rotational motion is less familiar to us since we generally tend to move in straight line segments rather than spinning around an axis. However, what we need to know in order to work problems in the rotational realm we have already seen in the linear realm.

For every linear variable, there is a corresponding angular variable. For every linear type of problem, there is a corresponding rotational type of problem. So rather than starting from scratch and developing new procedures to solve rotational problems, we can simply adapt our linear methodology to rotational problems.

As with linear motion, where there are three directions along which we can move, there are three directions around which we can rotate. However, where we for the most part limited our study of linear problems to two dimensions (primarily because it is easy to represent on a two dimensional piece of paper or screen), rotational motion will be limited to one dimension (primarily because it also is easy to represent on the same two dimensional piece of paper or screen – the rotational axis will be perpendicular to the plane of motion).

Technically, the direction of rotation is along the axis around which the motion occurs. So, for instance, a merry-go-round spinning in the $x - y$ plane will have a direction of rotation in the z-direction. However, at this level the direction is described as clockwise or counter-clockwise, and/or with a curved arrow showing the curved path taken.



In the image at right, the rotation direction is along the $+z$-direction, which is out of the screen using the right-hand-rule. (Direction vectors are straight lines, not curves – hence the direction is the axis about which it rotates.) The AP will designate the sense of rotation as **clockwise** or **counterclockwise** in the plane of the paper or screen as viewed from above.

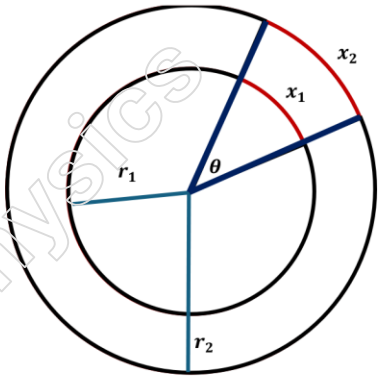
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Introduction to Rotational Motion

Introduction to Rotational Motion

Angular Displacement

Consider two points on a disk that rotates around its center. The two points lie along the same radius vector. The first point is $\boldsymbol{r_1}$ from the from the center, and the second point is $\boldsymbol{r_2}$ from the center. After the disk rotates through an angle θ, the first point has traveled an arc length $\boldsymbol{x_1 = r_1\theta}$, and the second point an arc length of $\boldsymbol{x_2 = r_2\theta}$.



The two points rotated through the same angular displacement $\boldsymbol{\theta}$, but traveled different linear distances $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ because of the different radial position from the center of the disk.

$$\boldsymbol{x = r\theta}$$

This relationship relates the arc length $\boldsymbol{r\theta}$ to the linear distance $\boldsymbol{x}$ traveled by a point a distance $\boldsymbol{r}$ from the center of rotation as the object rotates through an angular displacement $\boldsymbol{\theta}$.

Angle Representations:

If we look at the above equation closely, we should notice that both $\boldsymbol{x}$ and $\boldsymbol{r}$ are expressed in terms of distance units – meters for instance. For this equation to make sense dimensionally, the angle $\boldsymbol{\theta}$ is properly unitless.

The above equation is actually the definition of an angle: $\boldsymbol{\theta \equiv \frac{x}{r}}$. An angle is a ratio of distances- the arc length to the radius. When expressed as such, the representation of that angle is in radians. The other two common angular representations are degrees and revolutions.

$$\boldsymbol{2\pi\, radians = 360^\circ = 1\, revolution}$$

One can work problems with revolutions or degrees, as long as we do not try to connect the linear and the angular variables. For instance, if an object rotates with an angular speed of 45 rpm, we can determine how many revolutions it experiences in 3 seconds, for example. But if we want to know the linear path length, we must represent the angle in radians.

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Introduction to Rotational Motion

Angular Speed

Linear:	Angular:
Speed	**Angular Speed**
$speed = \frac{displacement}{time}$	$angular\ speed = \frac{angular\ displacment}{time}$
$v = \frac{x}{t}$	$\omega = \frac{\theta}{t}$

Since $x = r\theta$, then

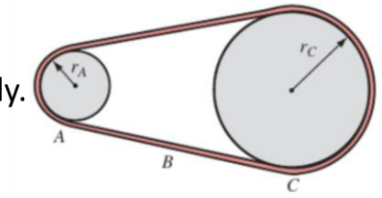
$$v = r\omega$$

All the discussion about the difference between instantaneous and average speeds apply here to angular speeds also.

All the graphical interpretations, **v = slope of $x - t$ graph** and **displacement = area under $v - t$ graph** applies here to the angular variables as well.

Example Problem:

Two wheels are coupled together by a fan belt. The radii of wheels A and B are $r_A = 12\ cm$ and $r_C = 24\ cm$ respectively. The point B on the belt has a constant speed of $v_B = 36\ \frac{cm}{s}$. Find ω_A, ω_C, and θ_A and θ_C in $5\ s$.



- The linear velocity for points on the outside of the wheel is $36\ \frac{cm}{s}$.
- So $v_B = r_A\omega_A = r_C\omega_C$. So $\omega_A = \frac{36}{12} = 3.0\ \frac{rad}{s}$ and $\omega_A = \frac{36}{24} = 1.5\ \frac{rad}{s}$.
- And $\theta_A = \omega_A t = (3.0)(5.0) = 15\ rad$ and $\theta_C = \omega_C t = (1.5)(5.0) = 7.5\ rad$.

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Introduction to Rotational Motion

Angular Acceleration

Linear:	Angular:
Acceleration	**Angular Acceleration**
$acceleration = \frac{\Delta(velocity)}{\Delta time}$	$angular\ acceleration = \frac{\Delta(angular\ speed)}{\Delta(time)}$
$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$

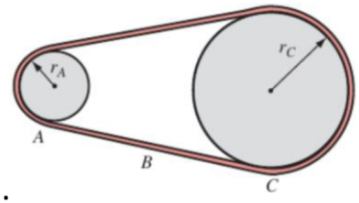
Since $v = r\omega$, then

$$a = r\alpha$$

All the discussion about the difference between instantaneous and average accelerations apply here to angular accelerations also.

All the graphical interpretations, **a = slope of $v - t$ graph** and **change in velocity = area under $a - t$ graph** applies here to the angular variables as well.

Example Problem:



Two wheels are coupled together by a fan belt. The radii of wheels A and B are $r_A = 12\ cm$ and $r_C = 24\ cm$ respectively. Starting from rest, the point B on the belt has a constant acceleration of $a_B = 36\ \frac{cm}{s^2}$. Find α_A, α_C, and ω_A and ω_C at $5\ s$.

- The linear acceleration for points on the outside of the wheel is $36\ \frac{cm}{s^2}$.
- So $a_B = r_A\alpha_A = r_C\alpha_C$. So $\alpha_A = \frac{36}{12} = 3.0\ \frac{rad}{s^2}$ and $\alpha_A = \frac{36}{24} = 1.5\ \frac{rad}{s^2}$.
- And $\omega_A = \alpha_A t = (3.0)(5.0) = 15\ \frac{rad}{s}$ and $\omega_C = \alpha_C t = (1.5)(5.0) = 7.5\ \frac{rad}{s}$.

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Rotational Kinematics

Rotational Kinematic Procedures

I have been striving to provide an efficient and powerful procedure for approaching any Physics problem in mechanics. For each of the kinematics, force, momentum, and energy units, I presented a single procedure that is very general, and consistent across topics. For example, choosing the positive direction to be the direction of the acceleration of the object maintains consistency across the coordinate systems used for kinematics and momentum applied to the same extended question prompt.

In each procedure, the first step has always been to make a sketch of the problem, which could be the kinematic diagram or the momentum diagram, for instance. This is always meant to provide consistency of the positive direction across different parts of an extended problem and the different vector equations involved. Often, I can see these diagrams in my head without committing them to paper or screen; however, if I want to guarantee that I make no mistake, I will always draw them out.

Now, in addition to consistency between the different linear coordinate systems (which is automatic if you follow the acceleration direction), we need consistency with the rotational coordinate system as well. Again this is easy if we can relate the linear acceleration direction and the angular acceleration, as we shall see later.

For the typical AP FRQ type question that involves, say, pulleys, torques, and masses moving along tables, then the linear kinematic/momentum diagrams will easily satisfy both. However, for the pure rotational kinematic diagram, for me at least, I get lost in the circle of motion – I am a linear thinker!

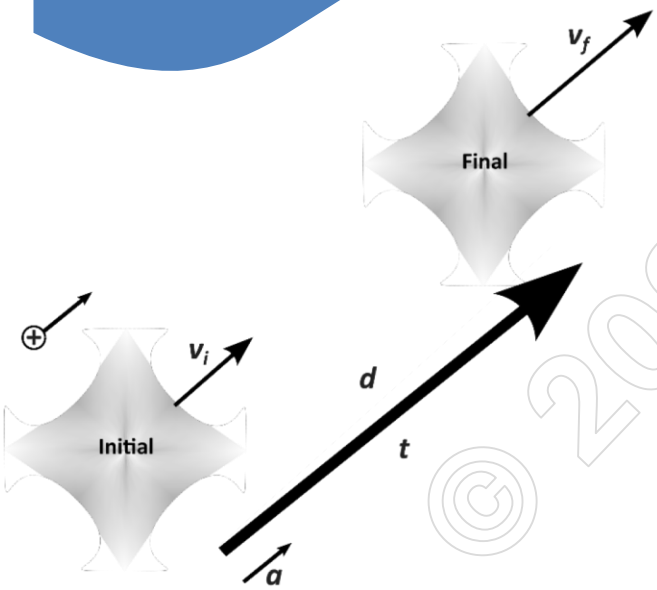
So, personally, I will translate any rotational kinematic problem into the corresponding 1-D linear problem, even so far as using the linear kinematic variables and equation and then translating the final result into the rotation answer.

Although I can write rotational analogues to the car-truck race problem or the Cheetah problem, they would never be physically appealing – who cares how how long it takes for two separate wheels to spin the same number of times. So the most complicated pure rotational kinematic problem will be an analogue to an object moving in a straight line undergoing pricewise constant angular accelerations.

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General Procedure for any Rotational Kinematic Problem



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General Procedure for any Rotational Kinematic Problem

1. **Unwrap the Rotational Kinematic Problem**
   - Translate into the analogue 1-D kinematic problem
2. **Draw Kinematic Diagram**
   - based on diagram at left
3. **Choose Naked Equation**
   - which of the four kinematic equations to use
4. **Dress Equation**
   - with variables in the diagram
   - values are positive or negative based on the choice of positive direction
5. **Solve**
   - if "best" equation was chosen, finished
   - if not, a second equation must be chosen - simultaneous equations

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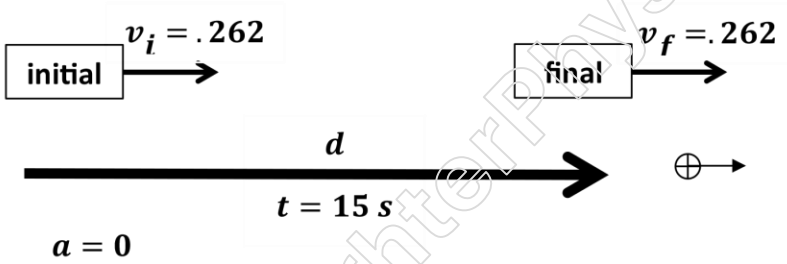
Rotational Kinematic Problem Examples - Example 1

Example 1:

A Ferris wheel is rotating at a constant $2.5\ rpm$. What is its angular displacement after $15\ s$?

$2.5\ rpm = 2.5\ \frac{rev}{min} = 0.262\ \frac{rad}{s}$

We need to make sure we use radians



$$d = \frac{1}{2}(v_f + v_i)t$$

$$d = \frac{1}{2}(.262 + .262)15$$

$$d = 3.93$$

Now translate back into rotational variables:

$$\theta = 3.93\ radians = 225° = 0.62\ rev$$

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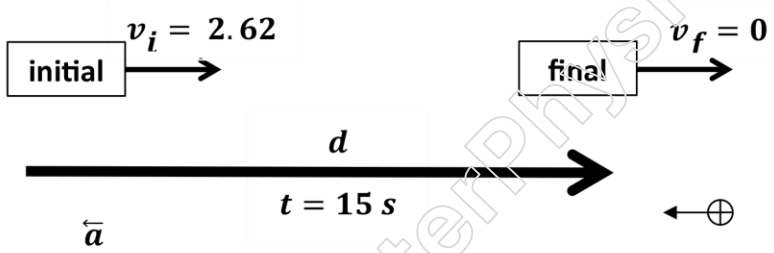
Rotational Kinematic Problem Examples - Example 2

Example 2:

A wheel is rotating at $25\ rpm$. A uniform braking force is applied to the wheel which stops in $15\ s$. What is the angular acceleration of the wheel?

$25\ rpm = 25\ \frac{rev}{min} = 2.62\ \frac{rad}{s}$

We need to make sure we use radians



$$v_f = v_i + at$$

$$0 = -2.62 + a15$$

$$a = 0.17$$

Now translate back into rotational variables:

$$\alpha = 0.17\ \frac{rad}{s^2} = 0.028\ \frac{rev}{s^2}$$

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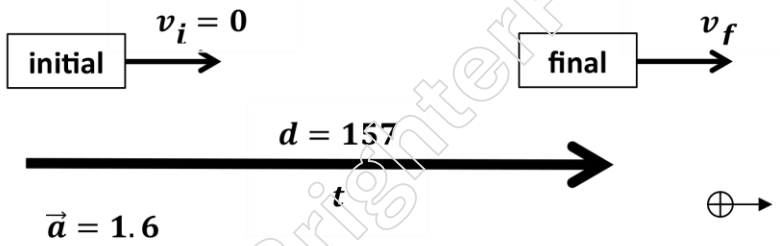
Rotational Kinematic Problem Examples - Example 3

Example 3:

Starting from rest, a wheel spins up at a rate of $1.6\ \frac{rad}{s^2}$. How long does it take for the wheel to rotate 25 times?

$25\ rev = 157\ rad$

We need to make sure we use radians



$$d = v_i t + \frac{1}{2}at^2$$

$$157 = (0)t + \frac{1}{2}(1.6)t^2$$

$$t = 196\ s$$

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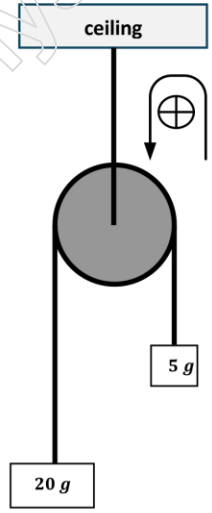
Rotational Kinematic Problem Examples - Example 4

Example 4:

The pulley system at right has an acceleration of $0.6\ \frac{m}{s^2}$. The pulley has a radius $r = 10\ cm$. The $5\ g$ mass is pulled downwards so that the pulley has an initial angular velocity of $12\ \frac{rad}{s}$ clockwise. Find the angular velocity of the pulley after the it has undergone a net angular rotation of 45 revolutions counterclockwise.

$$\alpha = \frac{a}{r} = \frac{.6}{.1} = 6.0\ \frac{rad}{s^2}$$

$$\theta = 45(2\pi) = 282.7\ rad$$

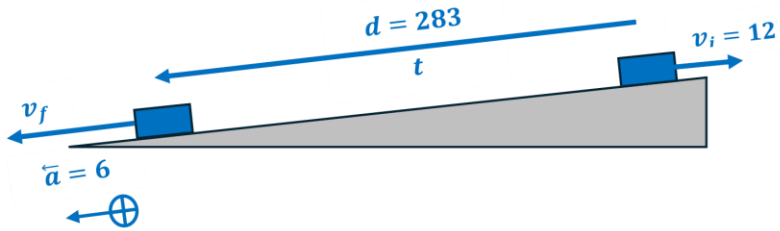


$$\omega_f^2 = \omega_i^2 + 2\alpha d$$

$$\omega_f^2 = (-12)^2 + 2(6)(282.7)$$

$$\omega_f = 59.5\ \frac{rad}{s}$$

Without the pulley reference and drawing above, the kinematic diagram below would be a good linear analogue to the rotation question presented.



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Rotational Kinematics Four Square - Part 1

1. Express the following angles in radians and revolutions. a) 45° b) 127° c) 445°	**2.** A ball rolls along a level floor. After 3.5 m, the ball has made 12.0 $rotations$. What is the radius and diameter of the ball?
3. A child sits 1.2 m from the center of a rotating merry-go-round that makes one revolution in 4.0 s. What is the linear speed of the child?	**4.** A child sits 1.2 m from the center of a rotating merry-go-round that makes one revolution in 4.0 s. What are the radial and tangential components of the acceleration of the child.

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Rotational Kinematics Four Square Part 1

Student Practice Blank

Rotational Kinematics Four Square - Part 2

1. A fan spins at a rate of $850\ rpm$. When switched off, it rotates $1250\ times$ before stopping. What is the angular acceleration of the fan?	**2.** A $31\ cm$ diameter wheel accelerates from $240\ rpm$ to $360\ rpm$ in $6.8\ s$. How far will a point on the edge of the wheel have traveled in this time?
3. A centrifuge accelerates from rest to $15000\ rpm$ in $240\ s$. Through how many revolutions did it rotate in this time?	**4.** A fan spins at a rate of $850\ rpm$. When switched off, it rotates $1250\ times$ before stopping. How long does it take for the fan to stop spinning?

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Rotational Kinematics Four Square Part 2

Student Practice Blank

Water Wheel Worksheet

A water wheel that is used to power a grist mill rotates *six times every minute*. The operator of the mill notices a problem with the mill and needs to stop the wheel to fix the issue. It takes the operator 45 *seconds* to reach the breaking system during which time the wheel continues to spin at a constant rate. Once applied, the braking system causes the wheel to change its rate of rotation by 0.25 $\frac{rad}{s^2}$. During the total time it takes for the operator to get to the braking system and bring the wheel to a full stop, how many rotations did the wheel make?



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Water Wheel Worksheet

Student Practice Blank

Linear to Rotational Conversion Table

Linear World	Rotational World	Ancillary Relations
time t	time t	
position x	angle (angular position) θ	$x = r\theta$ arc length
velocity v	angular velocity ω	$v_{tangential} = r\omega$
acceleration a	angular acceleration α	$a_{tangential} = r\alpha$
average velocity $v_{ave} = \Delta x/\Delta t$	average angular velocity $\omega_{ave} = \Delta\theta/\Delta t$	
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$	
$v_f^2 = v_i^2 + 2ad$	$\omega_f^2 = \omega_i^2 + 2\alpha d$	
$d = v_i + \frac{1}{2}at^2$	$\theta = \omega_i + \frac{1}{2}\alpha t^2$	
$d = \frac{1}{2}(v_f + v_i)t$	$\theta = \frac{1}{2}(\omega_f + \omega_i)t$	
		$a_{centr} \equiv v_{tang}^2/r = \omega^2 r$
		$\Sigma F_{radial} = m\, v_{tang}^2/r = m\omega^2 r$
mass (inertia) m	moment of inertia I	$I \equiv \Sigma mr^2 \equiv \int r^2 dm$
momentum $\vec{p} \equiv m\vec{v}$	angular momentum $\vec{L} = I\vec{\omega}$	$\vec{L} \equiv \vec{r} \times \vec{v}$
force $\Sigma\vec{F} = m\vec{a}\ (= d\vec{p}/dt)$	torque $\Sigma\vec{\tau} = I\vec{\alpha}\ (= d\vec{L}/dt)$	$\vec{\tau} \equiv \vec{r} \times \vec{F}$
impulse $\vec{F}_{net}\Delta t \equiv \Delta\vec{p}$	angular impulse $\vec{\tau}_{net}\Delta t \equiv \Delta\vec{L}$	
work $W_F = Fd\cos\theta$	work $W_\tau = \tau\theta$	$W_F \equiv \int \vec{F} \cdot d\vec{r}$ $W_\tau \equiv \int \vec{\tau} \cdot d\vec{\theta}$
kinetic energy $\frac{1}{2}mv^2$	kinetic energy $\frac{1}{2}I\omega^2$	$KE_{total} = KE_{linear} + KE_{rot}$

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