Projectile Motion

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Introduction to Projectile Motion

In this slide deck, a working model that can be used to solve any and all projectile motion problems is presented.

Technically, projectile motion is the 2-dimensional motion of an object that is subjected to a constant force. Because the force is constant, the acceleration is constant in the direction of the force (so the four kinematic equations are all valid in that direction), and the acceleration is zero in the direction perpendicular to the direction of the force (so the velocity in that direction is constant and the four kinematic equations truncate to one equation: $d = vt$).

The most obvious situation is therefore motion due to the Earth's gravitational force on an object near the Earth's surface. The acceleration is constant vertically and the velocity is constant horizontally.

It is not hard to imagine other possibilities. For instance, a charged particle moving through a constant electric field, or a marble launched along the surface of a slanted board. A sketch of these two scenarios are presented on the next slide.

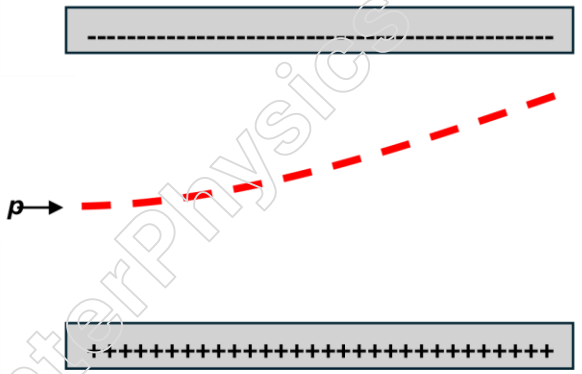
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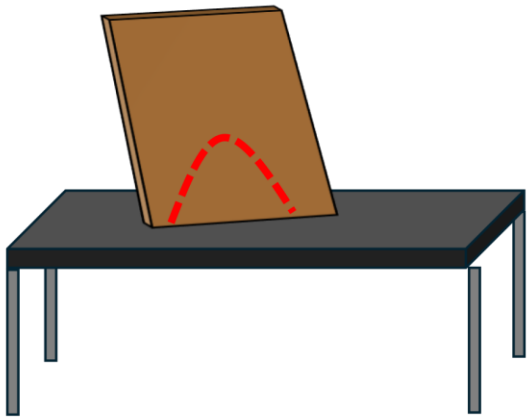
Introduction to Projectile Motion

Two Examples of Projectile Motion Not Due to Free-Fall

Trajectory of a positively charged proton with an initial horizontal velocity entering a region of a constant electric field due to the two oppositely charged plates. The electric field is upwards, and the gravitational field is downwards. For a large enough electric field, the net force is upwards.



An inclined plane is attached to a flat table. A marble is launched up the ramp at some angle. The net force is down the ramp. The resulting trajectory is shown in red. The velocity horizontally is constant. The acceleration of the marble is down the ramp and depends on the angle between the ramp and the table.

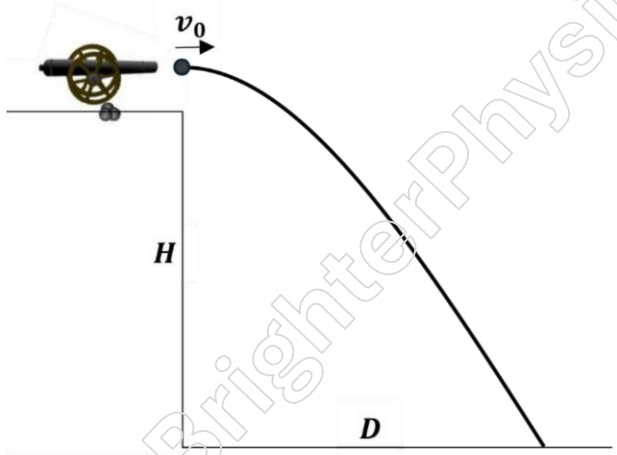


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Introduction to Projectile Motion

Let's consider a projectile motion problem – say a cannon ball fired horizontally with initial speed $\boldsymbol{v_0}$ from a cliff of height $\boldsymbol{H}$ and it lands a distance $\boldsymbol{D}$ from the base of the cliff.

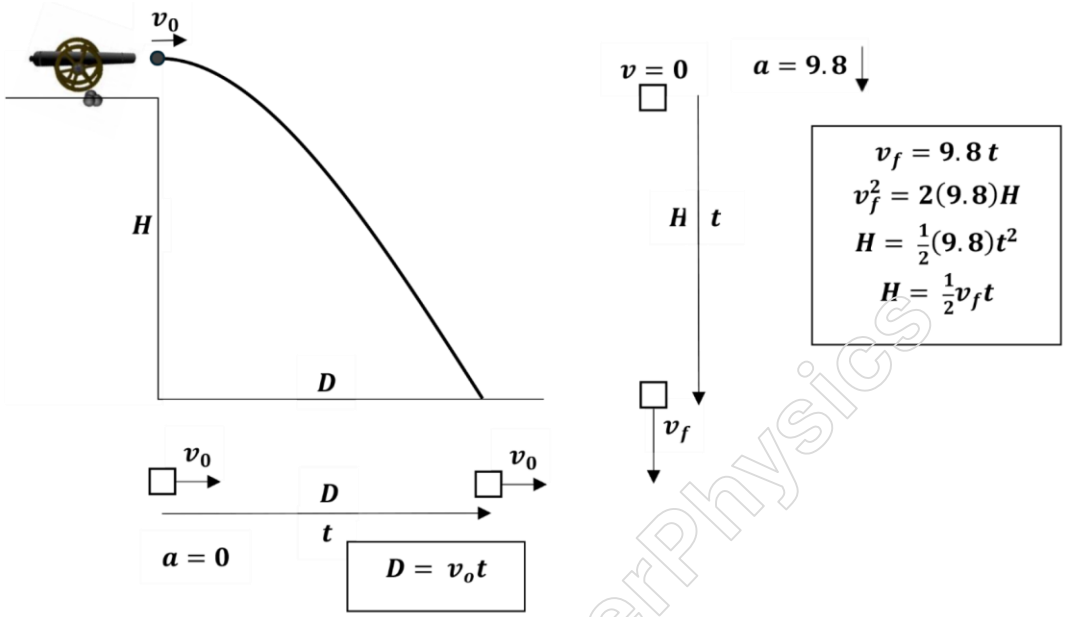


How can we attack this problem based on what we know about 1-D kinematics? Since this is two independent 1-D kinematic problems, that have the time $\boldsymbol{t}$ in common, we could make two kinematic diagrams, one for the horizontal motion and one for the vertical motion, and then solve them simultaneously.

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Introduction to Projectile Motion



To make this less crowded, only the dressed equations are shown. There is effectively only one equation for the horizontal. For the vertical, all four dressed equation are shown since no question has been asked.

Practically speaking, the third equation is, by far, the best equation to use since we are not usually interested in v_f, at least for introductory problems.

The positive direction has not been indicated here to lessen the crowding, although it should be. Positive is down and to the right.

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Introduction to Projectile Motion

Using this approach, notice that the horizontal and vertical problems are naturally separated from each other. In the horizontal problem, only the x-components (v_x) of the variables are used. In the vertical problem, only the y-components (a, $v_{o,y}$, and $v_{f,y}$) of the variables are used.

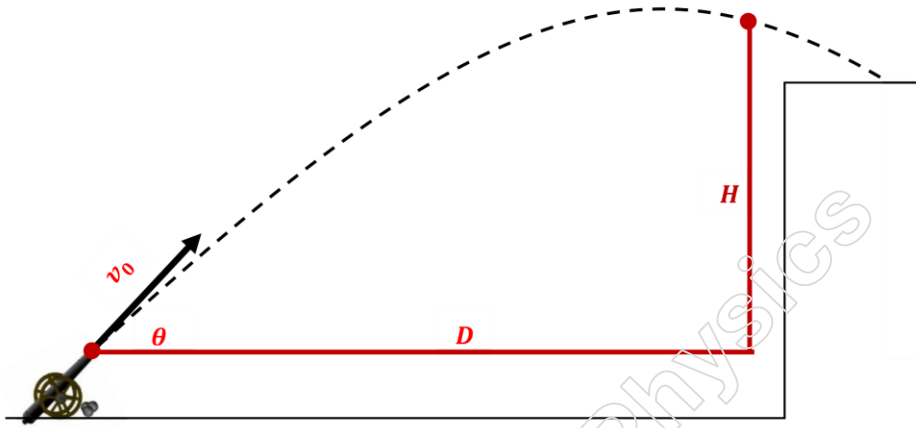
The time t is the same for both the horizontal and vertical problems – the vertical and horizontal motions start at the same time and end at the same time. We then typically have two simultaneous equations to solve in the end.

In more general projectile motion problems, the initial velocity is at an angle, generally measured from the horizontal. The speeds in the horizontal and vertical equations must then be the horizontal and vertical components of the velocity.

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Introduction to Projectile Motion



In the sketch above, a cannon fires a projectile up onto a cliff. To be clear, just as with 1-D kinematic problems, each kinematic equation involves ONLY the information from the initial and the final points of the motion.

Only the initial position and velocity of the projectile at the end of the cannon barrel and the position and velocity of the projectile at the position (D,H) relative to the cannon barrel are relevant. Where and when it reaches the topmost point where $v_y = 0$, or where and when it strikes the ground at the top of the cliff is not part of the problem.

For example, a question based on the above sketch might have several parts: Find the time until the projectile reaches the highest point of the trajectory, and find the horizontal distance traveled before it strikes the ground on top of the cliff. This would require two separate projectile motion diagrams and solutions.

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Introduction to Projectile Motion

Finally, most of us would write the gravitational field as $\boldsymbol{g} = 9.81\,\frac{m}{s^2}$. We know that this number can be determined assuming that the Earth is spherical and has a mass density that depends only on the radius. So the actual value depends on you position on the Earth.

In learning Physics, there is nothing to be gained by using $9.8\,\frac{m}{s^2}$, which is only an approximation anyway. We are lucky to live on a world where the value of $\boldsymbol{g}$ is very close to the very much "nicer" number $10\,\frac{m}{s^2}$. This is why the AP and IB both recommend that $g = 10\,\frac{m}{s^2}$ be used in free response questions on their exams.

Going forward, $\boldsymbol{g} = 10\,\frac{m}{s^2}$ will be used, which often results in more visually attractive numbers – "nicer" numerics.

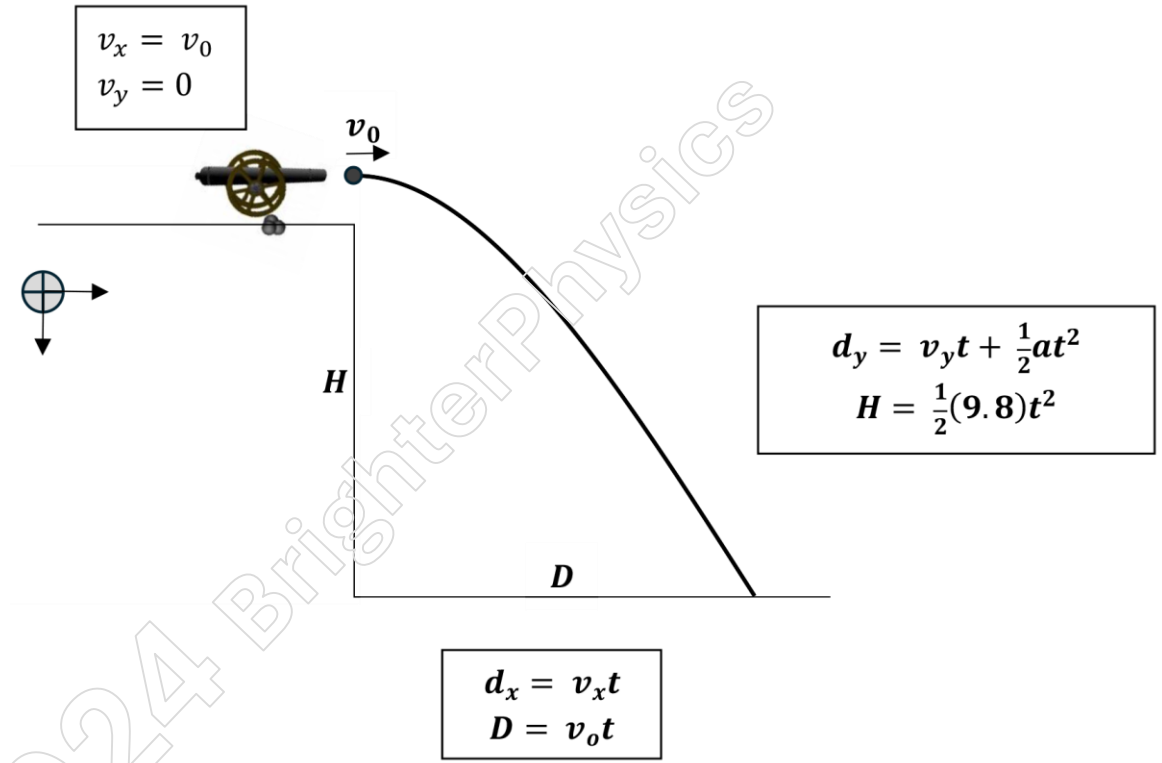
Of course, when comparing a calculation to experimental work, the correct value should be used.

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General Procedure for any Projectile Motion Problem

Cleaning up the previous crowded drawing, we have as a final projectile motion procedural diagram:



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General Projectile Motion Problem Procedure

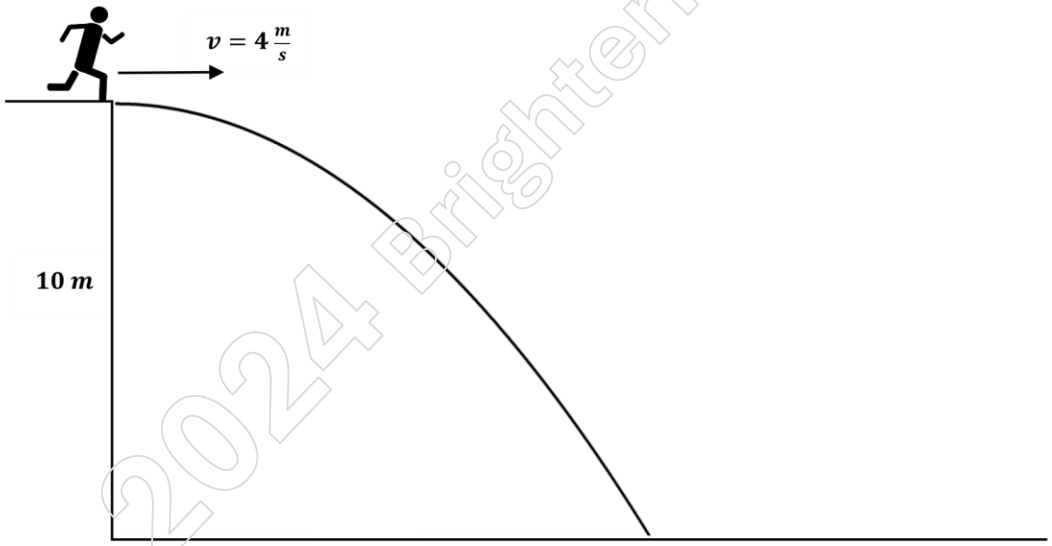
- Sketch (draw trajectory; attach kinematic variables)
- Find the x and y components of the initial velocity
- Choose positive direction
- Write down x-equation (only one choice here)
- Choose y-equation (typically the third equation)
- Solve

Projectile Motion Example I - Horizontal Launch

Phil the Fizziks Phool, runs off the edge of a 10 m tall horizontal diving platform. His speed when he leaves the platform is $4\ \frac{m}{s}$. How far does Phil move horizontally before he enters the water?

Step 1

Sketch of Problem



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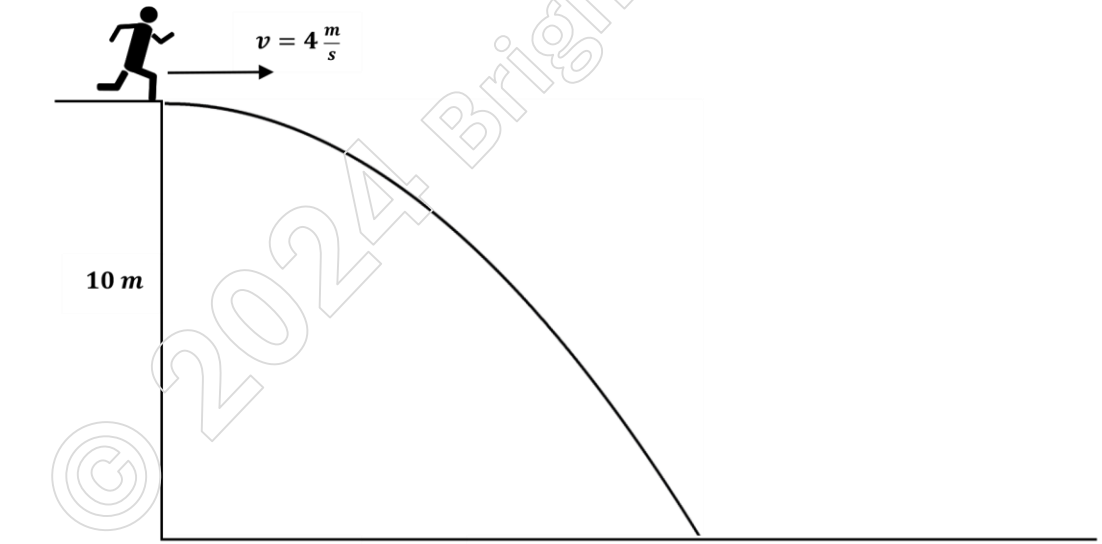
Projectile Motion Example I - Horizontal Launch

Phil the Fizziks Phool, runs off the edge of a 10 m tall horizontal diving platform. His speed when he leaves the platform is $4\ \frac{m}{s}$. How far does Phil move horizontally before he enters the water?

Step 2

Write Components of Velocity

$v_x = 4$
$v_y = 0$



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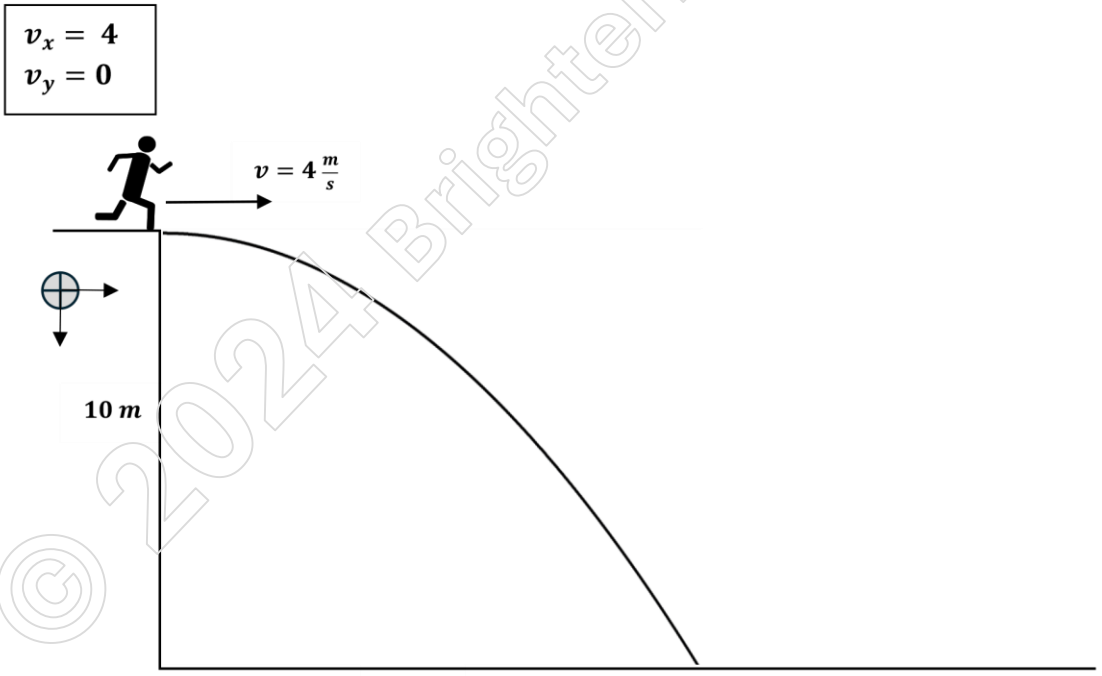
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Projectile Motion Example I - Horizontal Launch

Phil the Fizziks Phool, runs off the edge of a 10 m tall horizontal diving platform. His speed when he leaves the platform is $4\ \frac{m}{s}$. How far does Phil move horizontally before he enters the water?

Step 3

Choose Positive Direction



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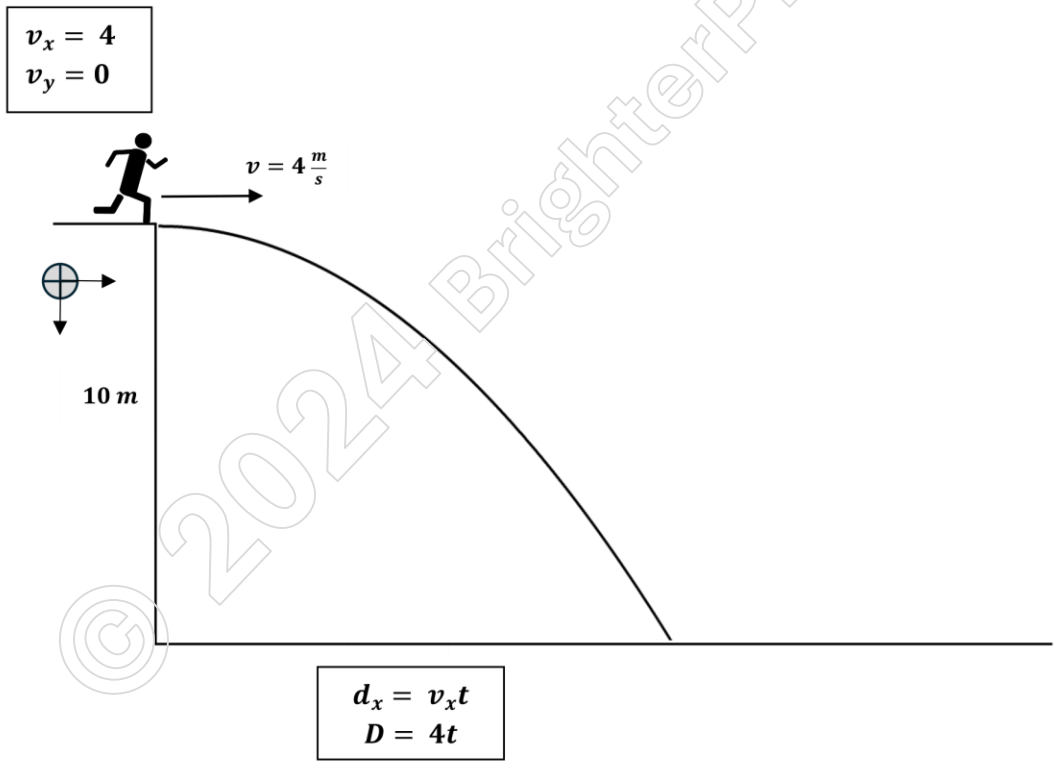
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Projectile Motion Example I - Horizontal Launch

Phil the Fizziks Phool, runs off the edge of a 10 m tall horizontal diving platform. His speed when he leaves the platform is $4\ \frac{m}{s}$. How far does Phil move horizontally before he enters the water?

Step 4

Choose the x-Equation



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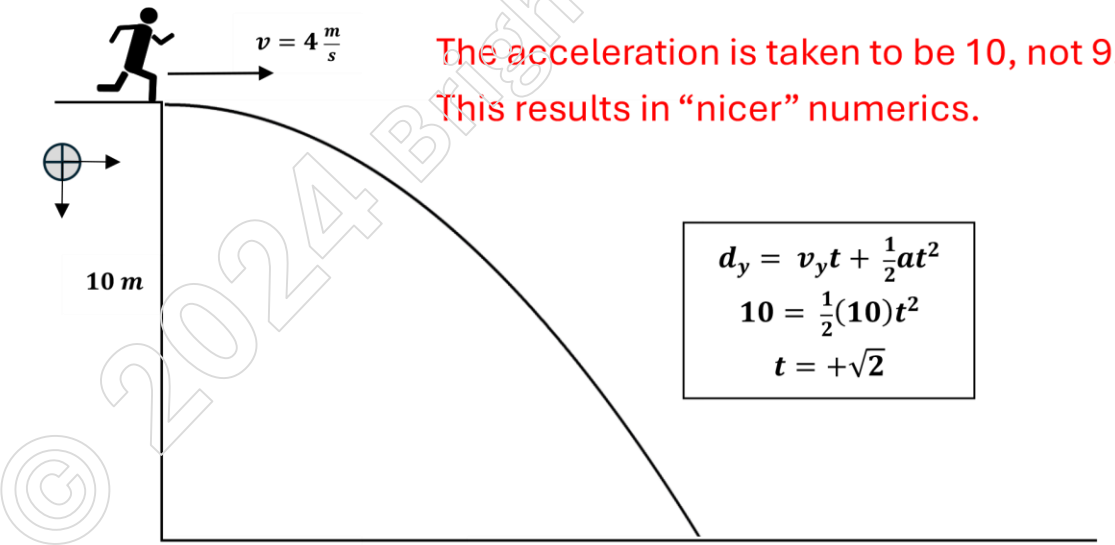
Projectile Motion Example I - Horizontal Launch

Phil the Fizziks Phool, runs off the edge of a 10 m tall horizontal diving platform. His speed when he leaves the platform is $4\ \frac{m}{s}$. How far does Phil move horizontally before he enters the water?

Step 5

Choose the y-Equation

$v_x = 4$
$v_y = 0$



The acceleration is taken to be 10, not 9.8. This results in "nicer" numerics.

$$d_y = v_y t + \frac{1}{2}at^2$$
$$10 = \frac{1}{2}(10)t^2$$
$$t = +\sqrt{2}$$

$$d_x = v_x t$$
$$D = 4t$$

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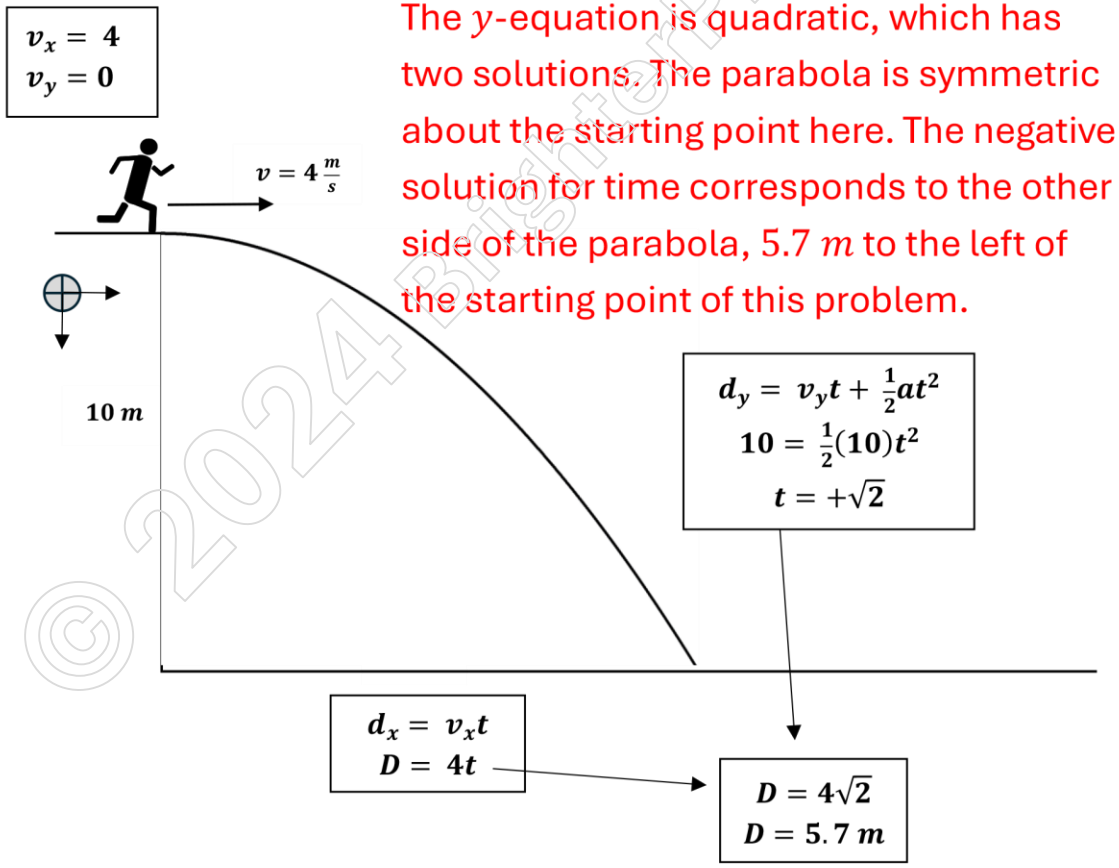
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Projectile Motion Example I - Horizontal Launch

Phil the Fizziks Phool, runs off the edge of a 10 m tall horizontal diving platform. His speed when he leaves the platform is $4\ \frac{m}{s}$. How far does Phil move horizontally before he enters the water?

Step 6

Solve the Simultaneous Equations



The y-equation is quadratic, which has two solutions. The parabola is symmetric about the starting point here. The negative solution for time corresponds to the other side of the parabola, $5.7\ m$ to the left of the starting point of this problem.

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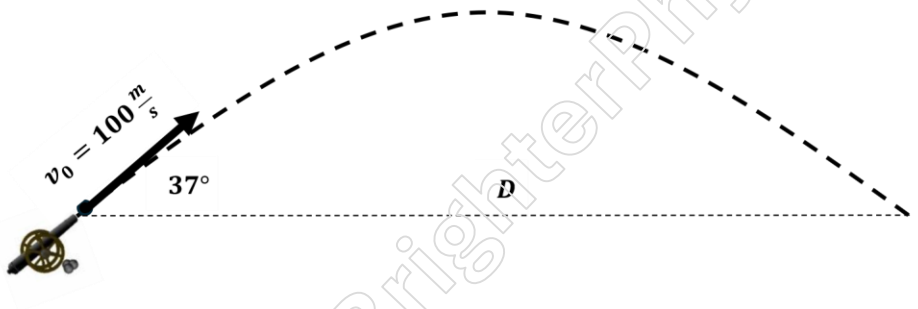
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Projectile Motion Example II - Ground To Ground

A cannonball is fired at 37° with a speed of 100 m/s.
How far does the projectile travel before striking the ground?

Step 1

Sketch



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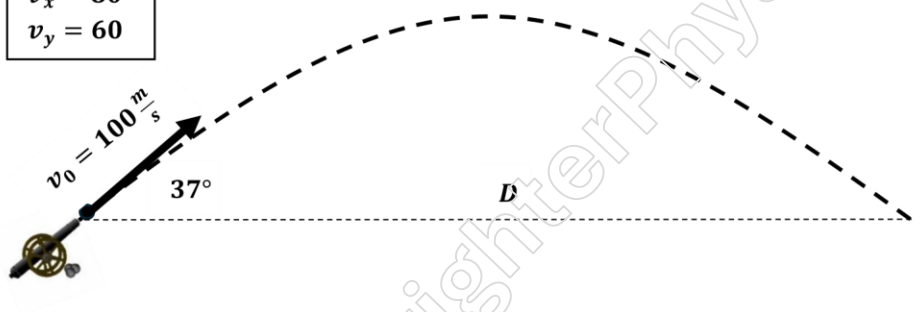
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Projectile Motion Example II - Ground To Ground

A cannonball is fired at 37° with a speed of 100 m/s.
How far does the projectile travel before striking the ground?

Step 2
Velocity Components

$v_x = 80$
$v_y = 60$



$$v_x = v_0 \cos 37° = 100(0.8)$$
$$v_y = v_0 \sin 37° = 100(0.6)$$

37° and 53° are the (approximate) angles of a 3-4-5 right triangle.
Use of these angles are common and result in "nicer" numerics.

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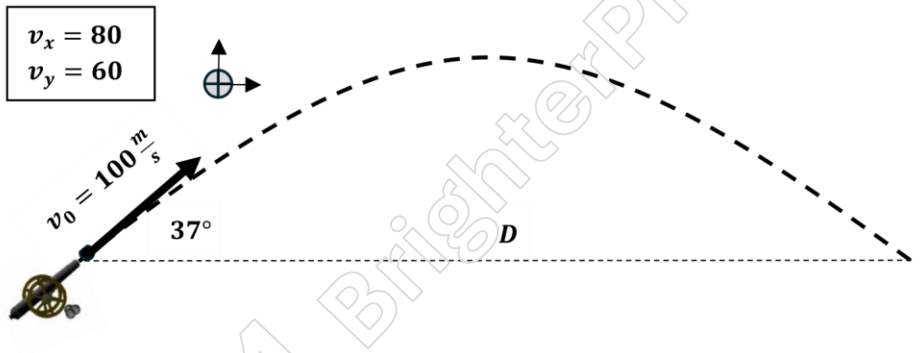
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Projectile Motion Example II - Ground To Ground

A cannonball is fired at 37° with a speed of 100 m/s.
How far does the projectile travel before striking the ground?

Step 3

Choose Positive



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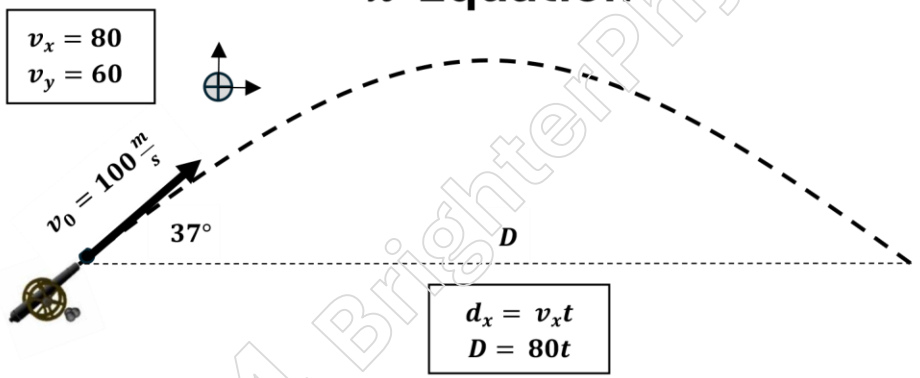
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Projectile Motion Example II - Ground To Ground

A cannonball is fired at 37° with a speed of 100 m/s. How far does the projectile travel before striking the ground?

Step 4

x-Equation



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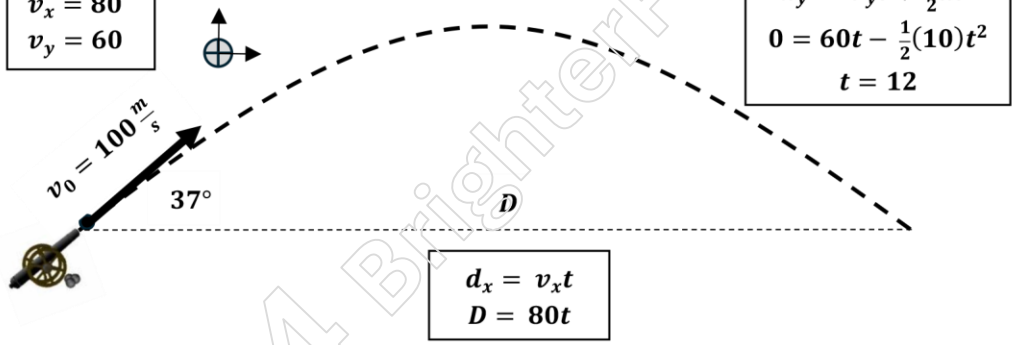
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Projectile Motion Example II - Ground To Ground

A cannonball is fired at 37° with a speed of 100 m/s.
How far does the projectile travel before striking the ground?

Step 5
y-Equation

Notice: v_y points upwards and a points downward - they have opposite signs in the equation.



The y-equation is quadratic, and therefore has two solutions.
The second solution is $t = 0$, which corresponds to the time of launch for which $D = 0$.
The acceleration is taken to be 10, not 9.8, for "nicer" numerics.

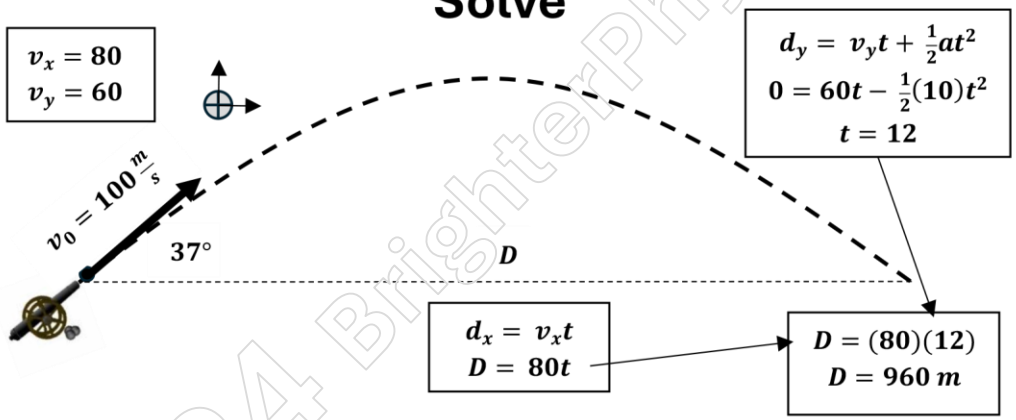
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Projectile Motion Example II - Ground To Ground

A cannonball is fired at 37° with a speed of 100 m/s.
How far does the projectile travel before striking the ground?

Step 6
Solve



The two resulting equations, one from x and one from y, are then solved simultaneously for whatever question is asked.

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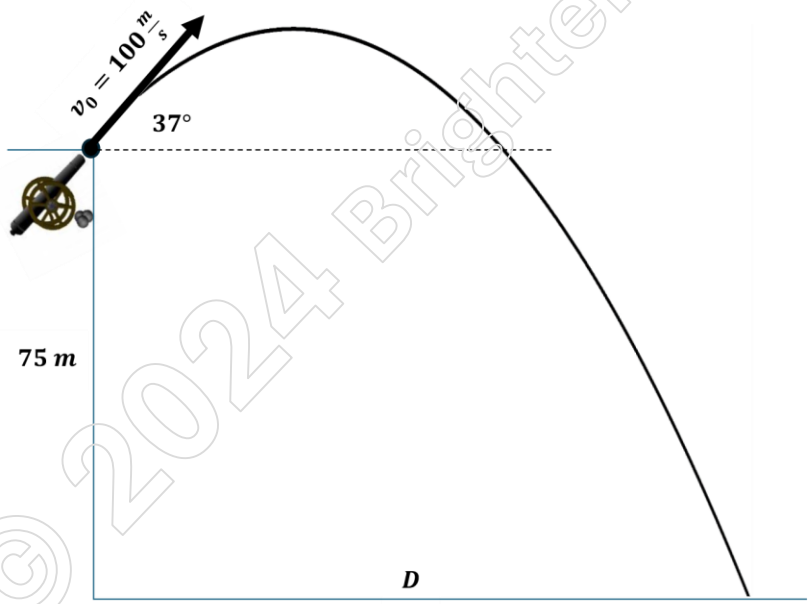
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Projectile Motion Example III - General Launch

A cannonball is fired at 37° with a speed of 100 m/s from the top of a 75 m tall cliff. How far does it travel before striking the ground?

Step 1

Sketch



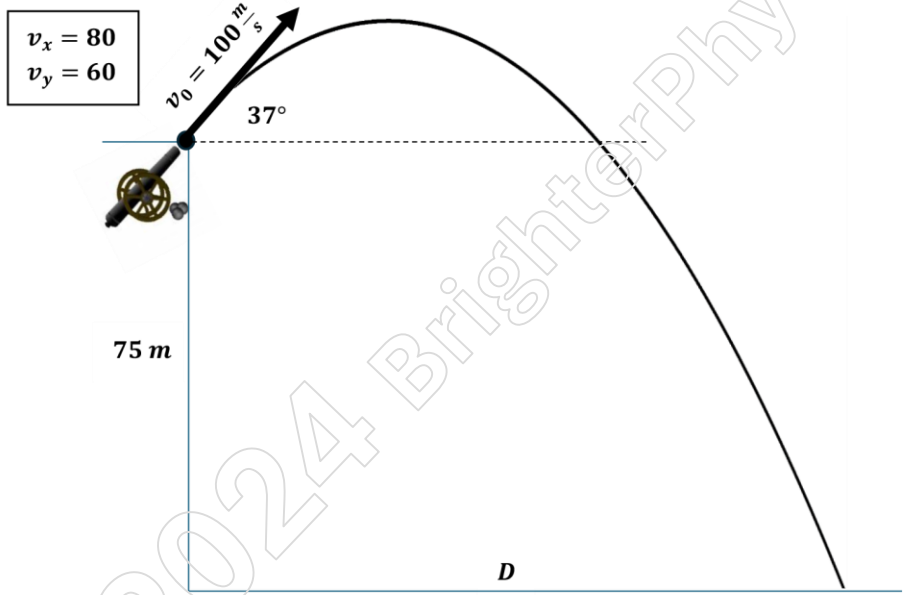
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Projectile Motion Example III - General Launch

A cannonball is fired at 37° with a speed of 100 m/s from the top of a 75 m tall cliff. How far does it travel before striking the ground?

Step 2
Velocity Components



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$$v_x = v_0 \cos 37° = 100(0.8)$$
$$v_y = v_0 \sin 37° = 100(0.6)$$

37° and 53° are the (approximate) angles of a 3-4-5 right triangle.

Use of these angles are common and result in "nicer" numerics.

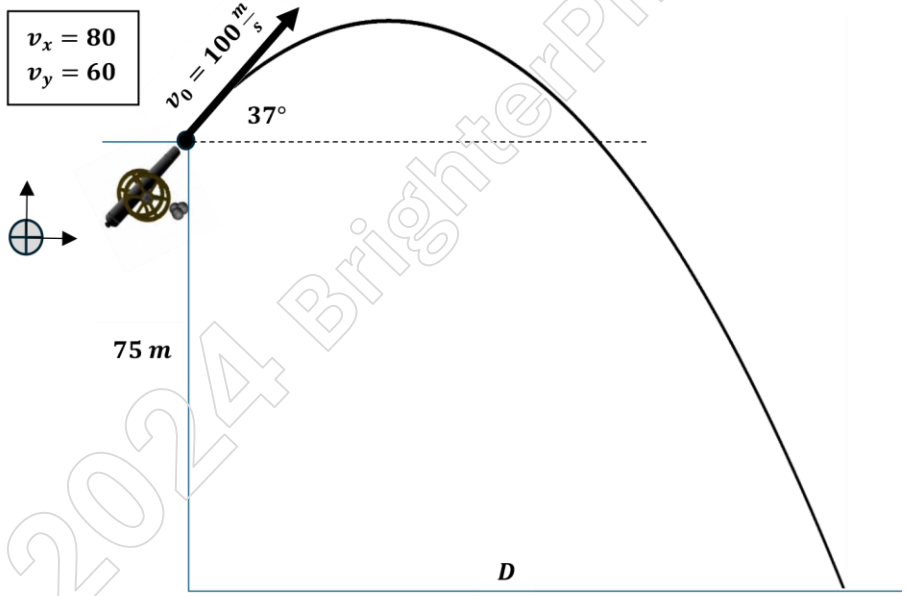
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Projectile Motion Example III - General Launch

A cannonball is fired at 37° with a speed of 100 m/s from the top of a 75 m tall cliff. How far does it travel before striking the ground?

Step 3

Choose Positive



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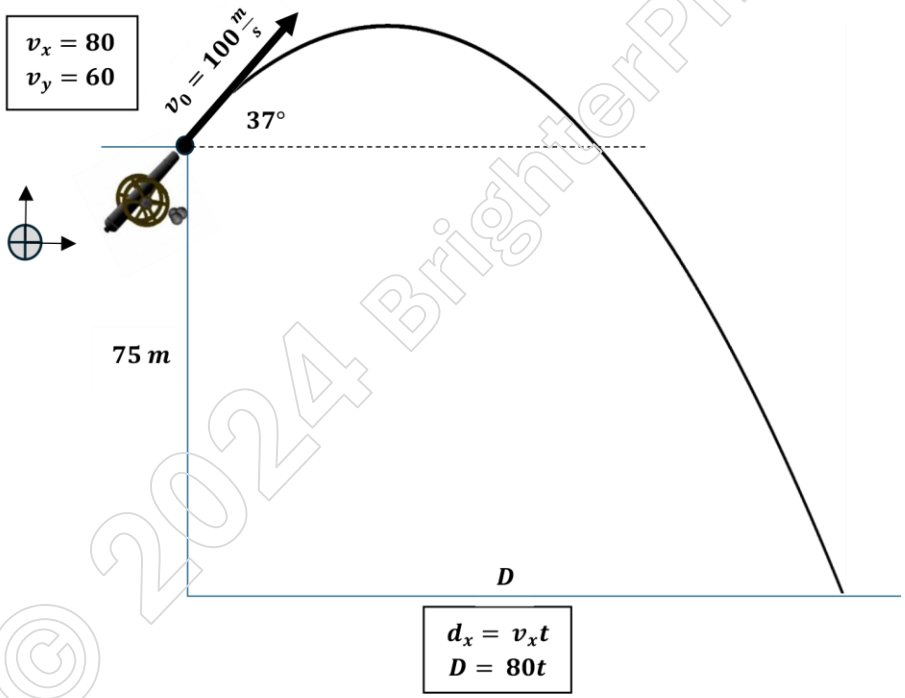
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Projectile Motion Example III - General Launch

A cannonball is fired at 37° with a speed of 100 m/s from the top of a 75 m tall cliff. How far does it travel before striking the ground?

Step 4

x-Equation



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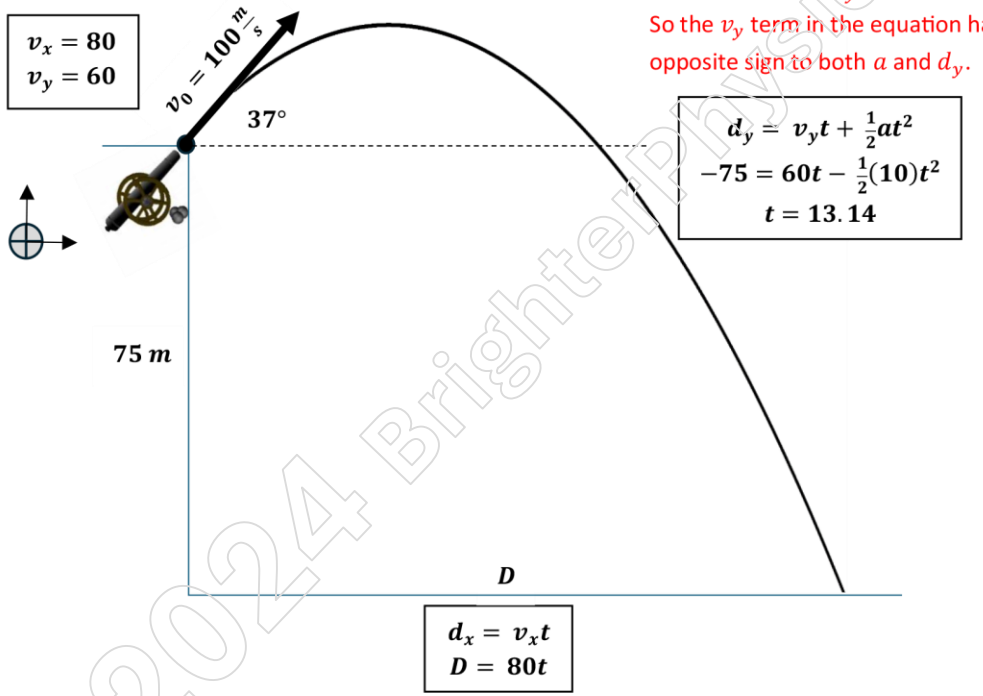
Projectile Motion Example III - General Launch

A cannonball is fired at 37° with a speed of 100 m/s from the top of a 75 m tall cliff. How far does it travel before striking the ground?

Step 5

y-Equation

Notice that both a and d_y point downward, while v_y points upwards. So the v_y term in the equation has opposite sign to both a and d_y.



The y-equation is quadratic, and therefore has two solutions. The other solution for time t is negative, which corresponds to before the time of launch and so is not physical. (We could extend the parabola backwards into the cliff to find the position.)

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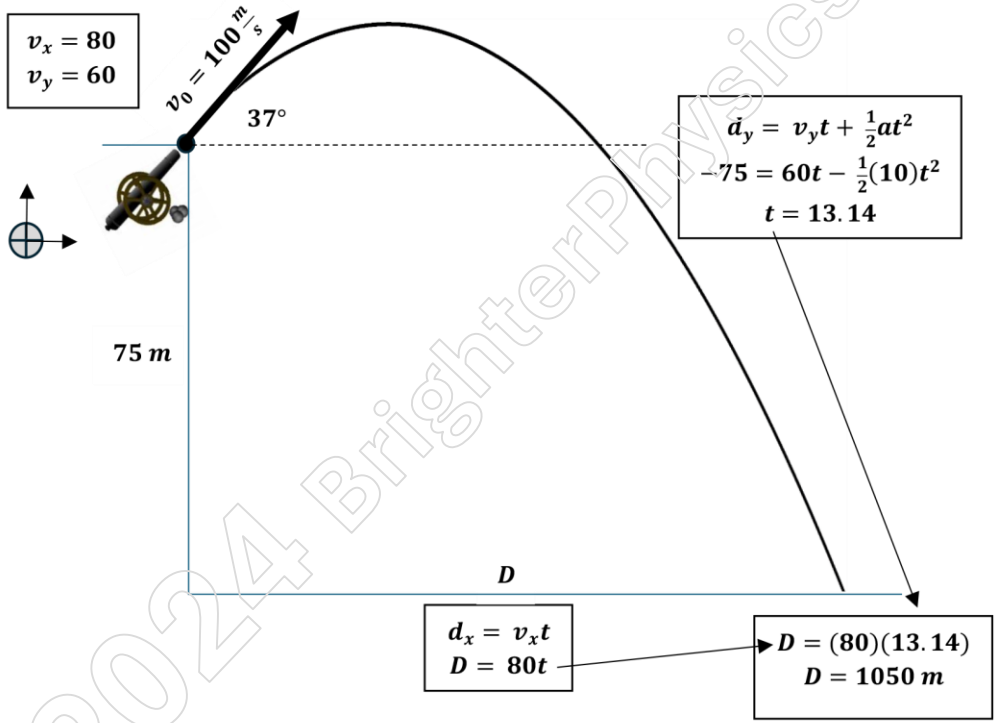
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Projectile Motion Example III - General Launch

A cannonball is fired at 37° with a speed of 100 m/s from the top of a 75 m tall cliff. How far does it travel before striking the ground?

Step 6

Solve



The two resulting equations, one from x and one from y, are then solved simultaneously for whatever question is asked.

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Comparing Projectile Motion Problems

Comparing the three projectile motion problem examples above, one should immediately notice that steps 1 through 4 are always the same. It is only in step 5 that the problems differ.

In all three problems shown the y-equation is quadratic, and so has two solutions. However, for the first two cases, the solution is simple because either the constant term or the term linear in time is zero, avoiding the need for the quadratic formula. It is unavoidable for the third case.

The difference, therefore, is not the complexity of the Physics. It is a question of the mathematical difficulty of dealing with the full quadratic equation.

Questions can also be written that involve the final velocity, specifically $v_{f,y}$. In that case, there are other three kinematic equations that can be considered – typically the second equation that is also quadratic. Again, the first two cases simplify – for horizontal launch problems, $v_{i,y} = 0$ as before, and for ground-to-ground problems, $v_{i,y} = -v_{f,y}$. It is only for the more general problems where the mathematics becomes more difficult. The difficulty of the Physics is the same in all cases.

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Projectile Motion Four Square Worksheet

Use the Projectile Motion Procedure to solve each of the following problems in the space provided.

1. A vindictive pigeon, flying horizontally at $20\ \frac{m}{s}$ at a height of $25\ m$ lets loose a dropping seeking revenge on a student who always chases him away. How far horizontally from the target should pigeon let loose?	**2.** An archer shoots an arrow with a speed of $50\ \frac{m}{s}$ at an angle of $30°$. How far does the arrow go before striking a target that is at the same height as the archer?
3. A pack of lemmings follow each other off a cliff. They are running at $3.892\ \frac{m}{s}$ when they run off the cliff. If the cliff is $12\ m$ high, how far from the cliff do the lemmings hit the ground?	**4.** A catapult launches a Monte Python cow at an angle of $37°$ from the horizontal. The catapult is on the top of a $200\ m$ tall cliff. The cow travels $1050\ m$ horizontally before striking the ground? Find the initial speed v_0 of the cow as it leaves the catapult.

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Projectile Motion Four Square

Student Practice Blank

Projectile Motion Graph Laboratory

In this lab exercise, you will make plot the trajectory of a projectile and superimpose the velocity vector of the projectile onto the trajectory.

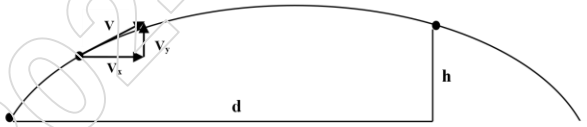
Assume:
Projectile has an initial velocity of 50 m/s making an angle of 53° with the horizontal. (Remember this is a special angle – use it as such!)

Find:

- Determine the height (h) and horizontal distance (d) from the launch point ***every 0.5*** seconds, starting from t = 0 through t = 8 s.
- Determine V_x, V_y, and V (x-component, y-component, and total velocity) ***every 1 s***, starting from t = 0 through t = 8 s.
- Complete the data table on the back side. Remember, there is velocity data only on the whole second.

Plot:

- Orient the paper so that the long dimension is horizontal. Place the origin three large blocks up from the bottom left corner.
- Using a scale of 1 cm = 10 m (1 block = 10 m), plot h against d for each of the 17 times. Draw a **SMOOTH** curve through the 17 points. (It should be parabolic).
- Using a scale of 0.5 cm = 10 m/s, plot V_x, V_y, and V, at each of the 9 points corresponding to every 1 s, starting from t = 0 through t = 8 s. (So for example, the y component of the velocity vector, $\mathbf{V_y}$, at t=2 will be a line 1.0 cm long (because V_y is 10 m/s).



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Projectile Motion Graph Laboratory

Student Practice Instructions

page 1

Projectile Motion Graph Laboratory

Data Table:

t (s)	d (m)	h (m)	v_x (m/s)	v_y (m/s)	v (m/s)
0.0					
0.5			n/a	n/a	n/a
1.0					
1.5			n/a	n/a	n/a
2.0					
2.5			n/a	n/a	n/a
3.0					
3.5			n/a	n/a	n/a
4.0					
4.5			n/a	n/a	n/a
5.0					
5.5			n/a	n/a	n/a
6.0					
6.5			n/a	n/a	n/a
7.0					
7.5			n/a	n/a	n/a
8.0					

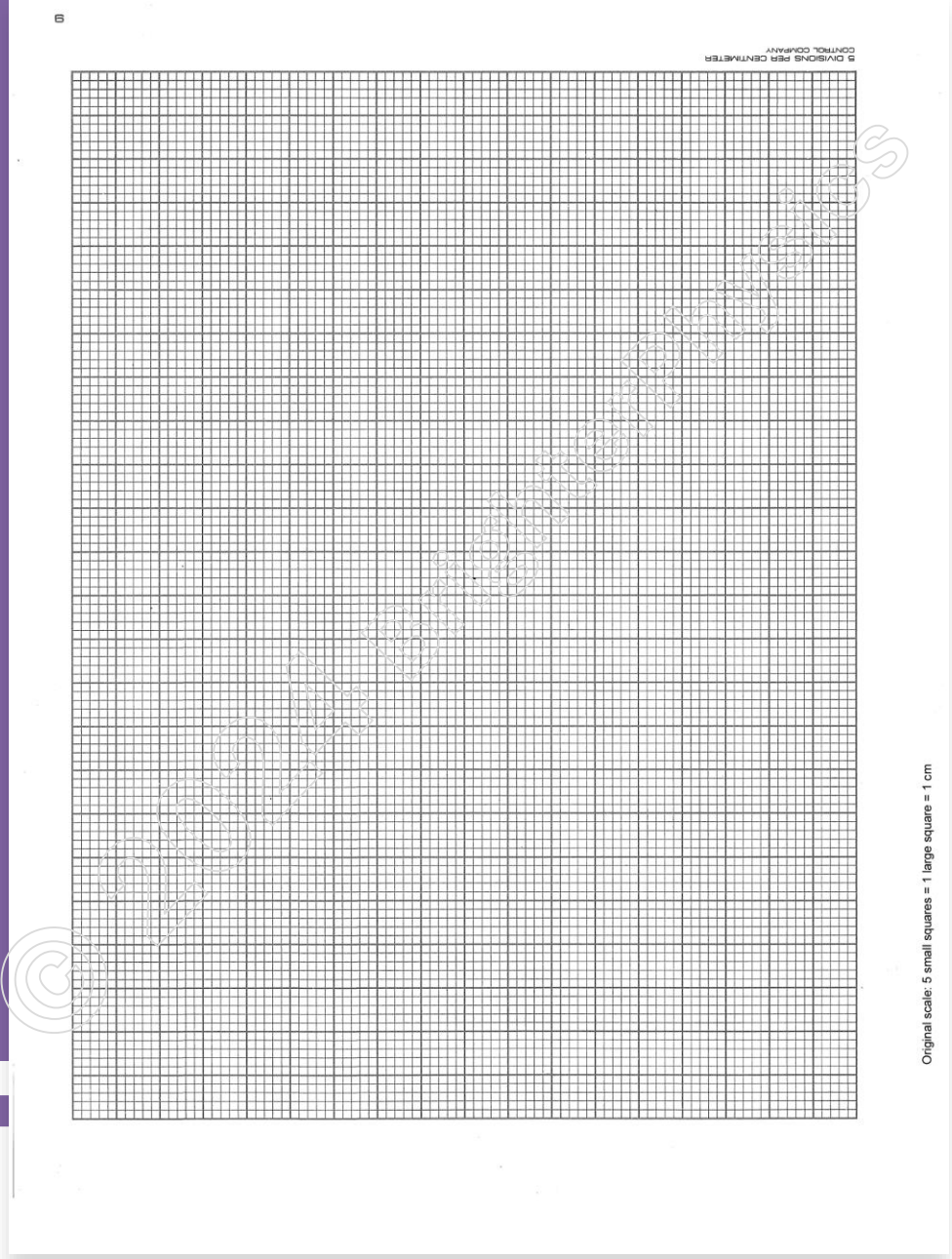
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Projectile Motion Graph Laboratory

Student Practice Blank Data Sheet

page 2



Projectile Motion Graph Laboratory

Student Practice Blank Graph Paper

page 2

Standard Projectile Motion Problems

The following three problems are examples of the standard projectile motion problems expected for AP level students.

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B29

A B29 Stratofortress flying at 400 km/hr and with an altitude of 5 km drops a weapon onto a target.

a) How far (horizontally) is the plane from the target when the weapon is released?

b) How long is the weapon in the air before it strikes the ground?

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B29

Student Practice
Blank

Howitzer

You are the firing officer for an artillery unit trying to destroy an enemy position 1.125 km away. The muzzle velocity of your howitzer is 0.15 km/s. At what angle(s) can you fire your gun and score a direct hit? [Hint: you will need the double angle trig identities at the end of the problem.]

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Howitzer

Student Practice
Blank

Napoleon Blonaparte

During the Battle of Waterloo, General Napoleon Blonaparte attempted to make culinary history by firing puff pastry at the British General Wellington. If you ignore air friction, what must be the muzzle velocity of the puff pastry if Wellington is on top of a 60 m tall cliff. Horizontally, Wellington is 1000 m from Napoleon's cannon? The cannon fires at an angle of 53° with respect to the horizontal.

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Napoleon Blonaparte

Student Practice Blank

Extension Projectile Motion Problems

The following four problems are examples of extension projectile motion problems for AP level students.

The same procedure is followed, but the problems have a twist: an extra object; other equations other than the third; unusual unknowns.

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Time and Height to Top

A soccer ball has an initial velocity of 35 $\frac{m}{s}$ at an angle of 75° from the horizontal when it is kicked and then lands on top of the school building. To what maximum height did the ball rise before landing on top of the building, and how long did it take to rise to that highest point?

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Time & Height to Top

Extension Problems

Student Practice Blank

Speed at Given Time

A soccer ball has an initial velocity of 35 $\frac{m}{s}$ at an angle of 37° from the horizontal when it is kicked onto the top of the school building. Find the speed of the soccer ball 1.5 and 3.5 seconds after it is kicked.

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Speed at Given Time

Extension Problems

Student Practice Blank

Angle of Impact

A high school Physics student throws a baseball with a speed of 25 $\frac{m}{s}$ and an angle of 37° from the horizontal off the top of a 8.5 m tall school building. At what angle, with respect to the horizontal, will the baseball strike the ground?

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Angle of Impact

Extension Problems

Student Practice Blank

Angle of Idiocy

Believe it or not!

In a recent game, a backup football player punted the football. When they kicked the football, they were at rest. Immediately after the kick, they accelerated uniformly at 1.5 m/s^2 downfield in the direction of the ball. A few seconds later the football hit them directly on the helmet (which we shall assume is the same height as the point of contact of the kick.) What was the angle of the kick, θ, which we will call the angle of idiocy.

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Angle of Idiocy

Extension Problems

Student Practice Blank

The Kinematic Equations

The Kinematic Equations

For an object with constant acceleration:

I. $v_f = v_i + at$

II. $v_f^2 = v_i^2 + 2ad$

III. $d = v_i t + \frac{1}{2}at^2$

IV. $d = \frac{1}{2}(v_f + v_i)t$

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a = constant acceleration during the time interval

t = time interval for the motion

d = displacement of object during time interval t

v_i = initial velocity during the time interval

v_f = final velocity during the time interval

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