Oscillations

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Oscillations

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Introduction to Oscillations

What is an Oscillation?

An oscillation is any motion that repeats itself, if we wait long enough. For AP Physics 1, this is restricted to the oscillatory motion of pendula, which can be swinging or torsional, and the oscillatory motion of masses attached to springs.

These two motion can be considered Harmonic since the resulting motion is has a regularly repeating oscillation about an equilibrium position.

We will be looking at specifically what is defined as Simple Harmonic Motion, SHM for short. This is a very specific motion where the motion variable as a function of time, $x(t)$ for instance, is sinusoidal in time.

It should be noted that there is a formal distinction between Harmonic motion and Simple Harmonic Motion, although a number of sources do not make the distinction. For instance, when Khan Academy refers to Harmonic Motion, they are referring to SHM.

There is a difference – a pendulum undergoes Simple Harmonic Motion only if the release angle is very small, whatever that means. If the angle is not small, then it does not undergo SHM, although it will still be Harmonic in that it has a period.

In short, an object undergoes Simple Harmonic Motion if its position variable as a function of time $x(t)$ is sinusoidal. So $x(t) = A\cos(\omega t + \delta)$. This can be connected to an object undergoing uniform circular motion. For an object undergoing uniform circular motion in the xy-plane, its $(x(t), y(t))$ coordinates as a function of time can be written as $(A\cos(\omega t + \delta), A\sin(\omega t + \delta))$. So mathematically, SHM is the 1-D shadow of 2-D uniform circular motion. The variable δ is the starting point in the uniform circular motion. The following link is to a visually appealing demonstration of this from the Harvard Natural Science Lecture Demonstrations entitled Circular Motion vs Mass on a Spring:



https://youtu.be/ZlleypTKfGY?si=f0qz3V6wzChYD4DK

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Introduction to Oscillations

Simple Harmonic Oscillation

Let's look at the consequence of the restriction to sinusoidal motion:

If $x(t) = A\cos(\omega t)$, then $v(t) = -A\omega\sin(\omega t)$, and then $a(t) = -A\omega^2\cos(\omega t)$. Since $F(t) = ma(t)$, then $F(t) = -m\omega^2 x(t)$.

So, an object will undergo Simple Harmonic Motion iff the force driving the motion is a restoring force that is linear in the displacement:

$$\boldsymbol{F(t) \propto -x(t)}$$

Hooke's Law Springs:

Ideal Springs, springs that obey Hooke's Law $F = -kx$, will undergo Simple Harmonic Motion if an attached mass is given some intitial displacement and released. The period $\boldsymbol{T}$ of the resulting motion is $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}$.

This can be obtained by equating the two expressions for the force: $\boldsymbol{F = -kx = -m\omega^2 x}$, so $\boldsymbol{k = m\omega^2}$. Remember what $\boldsymbol{\omega}$ is; it is the angular speed of the corresponding uniform circular motion. So substituing $\boldsymbol{\omega = \frac{2\pi}{T}}$ into $k = m\omega^2$ results in the expression for the period above.

Alternatively, if $\boldsymbol{T}$ is the time for one complete cycle of motion, then $\boldsymbol{x(t) = x(t+T)}$ and $\boldsymbol{v(t) = v(t+T)}$ must both be true so that both position and velocity repeat. So $\boldsymbol{\cos(\omega t) = \cos\big(\omega(t+T)\big)}$ and $\boldsymbol{\sin(\omega t) = \sin\big(\omega(t+T)\big)}$ which means that $\boldsymbol{\omega T = 2\pi}$, which can be shown using trig double angle formulae.

For completeness, we can look at a torsional spring: an object will undergo Simple Harmonic Motion iff the torque driving the motion is a restoring torque that is linear in the angular displacement:

$$\boldsymbol{\tau(t) = -\kappa\theta(t)}$$

Here, to avoid confusion between the angular velocity of the spring, $\omega(t)$, and the angular speed of the uniform circular motion, $\boldsymbol{\omega}$, I have used $\frac{2\pi}{T}$ instead. If $\theta(t) = \Theta\cos\left(\frac{2\pi}{T}t\right)$, then $\omega(t) = -\Theta\frac{2\pi}{T}\sin\left(\frac{2\pi}{T}t\right)$, and $\alpha(t) = -\Theta\left(\frac{2\pi}{T}\right)^2\cos\left(\frac{2\pi}{T}t\right)$. Since $\tau(t) = I\alpha(t)$, then $\tau(t) = -I\left(\frac{2\pi}{T}\right)^2\theta(t)$. Thus $\boldsymbol{\tau = -\kappa\theta = -I\left(\frac{2\pi}{T}\right)^2\theta}$. Following the same procedure as above for an ideal spring, the period of a torsional pendulum is given by $\boldsymbol{T = 2\pi\sqrt{\frac{I}{\kappa}}}$.

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Introduction to Oscillations

Simple Harmonic Oscillation

Pendulum:

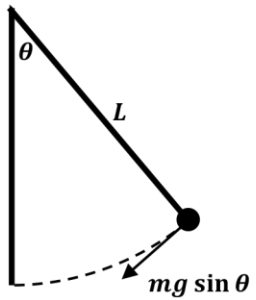
For a simple pendulum, a mass m hangs from a light string of length L, is pulled back and released. At some time t the position of the mass is shown in the figure below.

When the mass makes an angle θ with the vertical, the angular position of the mass is the angle θ and the torque acting on the mass is given by $\tau = -mgL\sin\theta$. So the torque is then proportional to $\sin\theta$, not θ.

However, for small angles, $\theta \ll 1$ rad, $\sin\theta \approx \theta$. In this small angle approximation case, $\tau \approx -mgL\theta$, in which case the result of the torsional spring can simply be coopted here.

Therefore, for small angles:

$$T = 2\pi\sqrt{\frac{I}{\kappa}} = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$



How does one quantify smallness as in $\theta \ll 1$? It is really an experimental statement based on the precision of whatever measuring devices are used.

Experimentally, one would measure the period as a function of θ, looking for the beginning of the deviation from a constant. If this experiment was done using a stopwatch to measure the time for one oscillation, then that deviation might begin to occur at around $60°$. If the period was determined by finding the time for 10 oscillations, then the deviation might begin at around $45°$. If we step away from humans doing the measurement and instead use something like Vernier photogates, the deviation might be driven down to about $3°$.

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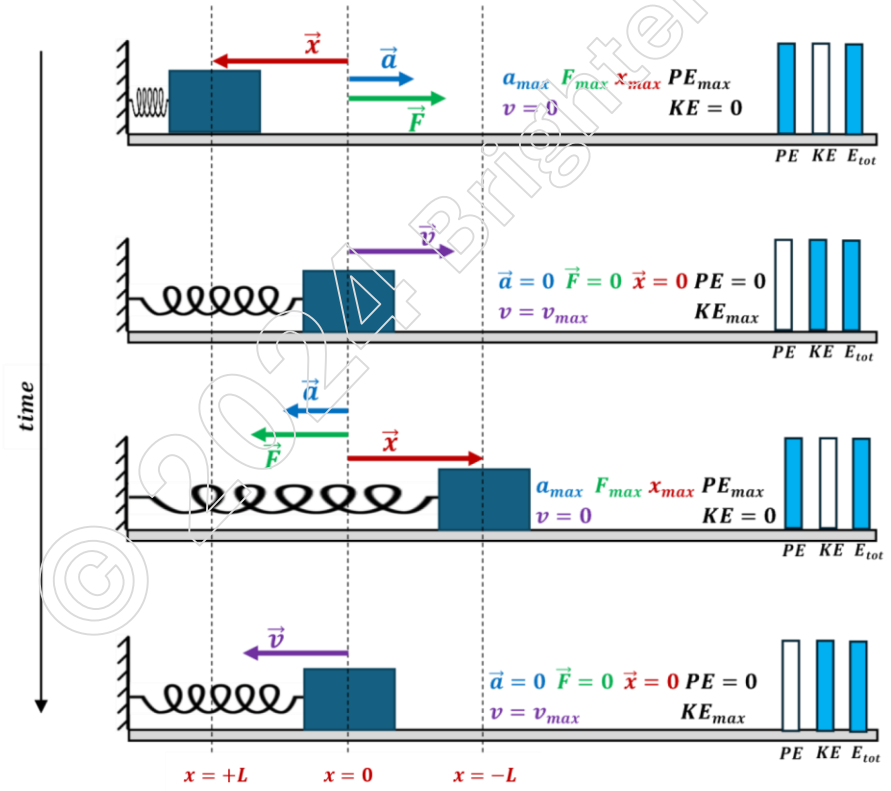
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Representations of SHM

Let's look at different ways to visualize SHM. It is simplest to draw, and most comfortable to use, a spring system. Suppose we have a mass $\boldsymbol{m}$ attached to a spring having spring constant $\boldsymbol{k}$ and pulled left to an initial position $\boldsymbol{x = +L}$ and released. We have seen that in this case $\boldsymbol{\omega = \sqrt{\frac{k}{m}}}$ and $\boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}$. Then $\boldsymbol{x(t) = L\cos\left(\frac{2\pi}{T}t\right)}$, $\boldsymbol{v(t) = -v_{max}\sin\left(\frac{2\pi}{T}t\right)}$, where $\boldsymbol{v_{max} = L\omega = \frac{2\pi L}{T}}$, and then $\boldsymbol{a(t) = -a_{max}\cos\left(\frac{2\pi}{T}t\right)}$, where $\boldsymbol{a_{max} = L\omega^2 = L\left(\frac{2\pi}{T}\right)^2}$.

We also have, since $KE = \frac{1}{2}mv^2$, $\boldsymbol{KE(t) = \frac{1}{2}m\,{v_{max}}^2\sin^2\left(\frac{2\pi}{T}t\right)}$, and since $PE = \frac{1}{2}kx^2$, we also have $\boldsymbol{PE(t) = \frac{1}{2}k\,L^2\cos^2\left(\frac{2\pi}{T}t\right)}$. Note that $\boldsymbol{m\,{v_{max}}^2 = m\,L\omega = k\,L^2}$, so therefore $\boldsymbol{ME(t) = KE(t) + PE(t) = \frac{1}{2}k\,L^2 = \frac{1}{2}m\,{v_{max}}^2}$ is constant.

Variable values at $\boldsymbol{t = 0}$, $\boldsymbol{\frac{1}{4}T}$, $\boldsymbol{\frac{1}{2}T}$, $\boldsymbol{\frac{3}{4}T}$:



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Representations of SHM

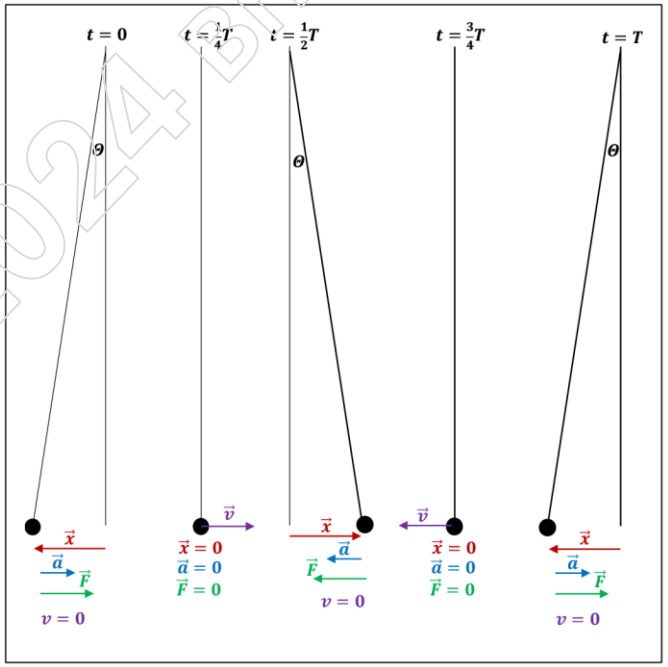
Representations of SHM

We can do something similar for a pendulum system, although it is not usually presented. It is more difficult to draw compared to the spring case.

Suppose we have a mass $\boldsymbol{m}$ attached to a massless string of length $\boldsymbol{L}$ and pulled left to an initial small angle $\boldsymbol{\Theta}$ and released. We have seen that in this case $\boldsymbol{T = 2\pi\sqrt{\frac{L}{g}}}$. Then $\boldsymbol{\theta(t) = \Theta\cos\left(\frac{2\pi}{T}t\right)}$, $\boldsymbol{\omega(t) = -\omega_{max}\,\Theta\sin\left(\frac{2\pi}{T}t\right)}$, where $\boldsymbol{\omega_{max} = \frac{2\pi}{T}}$, and then $\boldsymbol{\alpha(t) = -\alpha_{max}\,\Theta\cos\left(\frac{2\pi}{T}t\right)}$, where $\boldsymbol{a_{max} = \left(\frac{2\pi}{T}\right)^2}$.

We also have, since $KE = \frac{1}{2}I\omega^2$, $\boldsymbol{KE(t) = \frac{1}{2}I\,\omega_{max}{}^2\,\Theta^2\sin^2\left(\frac{2\pi}{T}t\right)}$. We also know $PE = mgh$. For a simple pendulum, $h = L(1-\cos\theta)$. In the small angle approximation, $\cos\theta \approx \frac{\theta^2}{2}$, and so $\boldsymbol{PE(t) = \frac{1}{2}mgL\Theta^2\cos^2\left(\frac{2\pi}{T}t\right)}$. Note that $\boldsymbol{I\,\omega_{max}{}^2 = mL^2\left(\frac{2\pi}{T}\right)^2 = mL^2\left(\frac{2\pi}{2\pi\sqrt{L/g}}\right)^2 = mgL.}$

Therefore $\boldsymbol{ME(t) = KE(t) + PE(t) = \frac{1}{2}mgL\Theta^2 = \frac{1}{2}I\,\omega_{max}{}^2\Theta^2}$ is constant.



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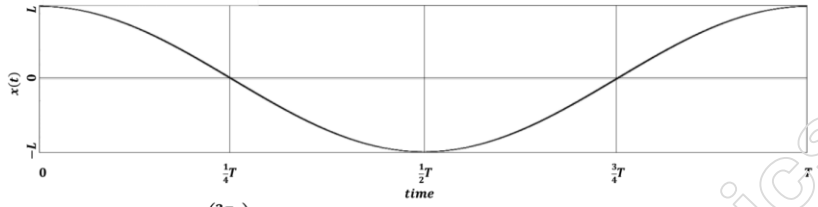
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Representations of SHM

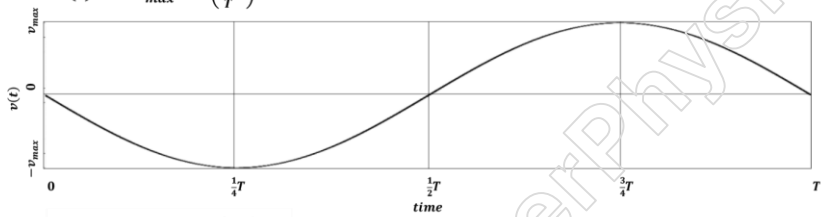
Representations of SHM: Stretch Start

Variables as a function of time for 1 period

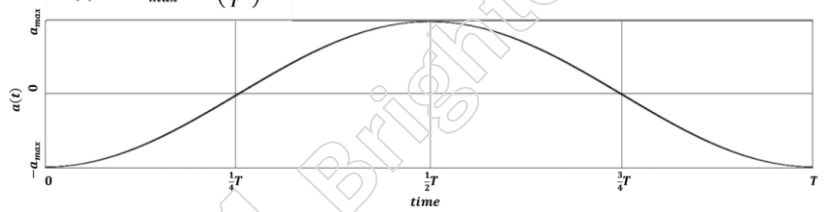
$x(t) = L\cos\left(\frac{2\pi}{T}t\right)$



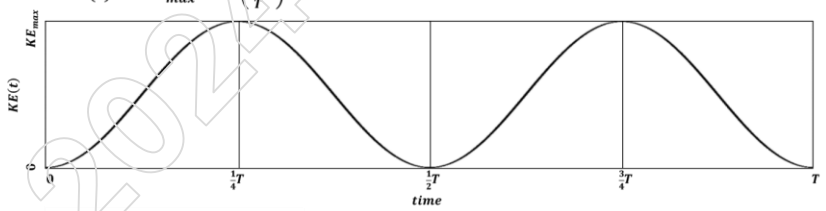
$v(t) = -v_{max}\sin\left(\frac{2\pi}{T}t\right)$



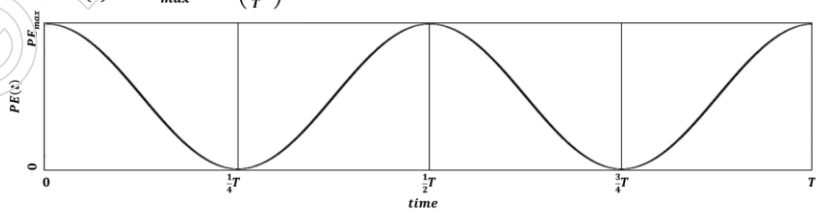
$a(t) = -a_{max}\cos\left(\frac{2\pi}{T}t\right)$



$KE(t) = KE_{max}\sin^2\left(\frac{2\pi}{T}t\right)$



$PE(t) = PE_{max}\cos^2\left(\frac{2\pi}{T}t\right)$



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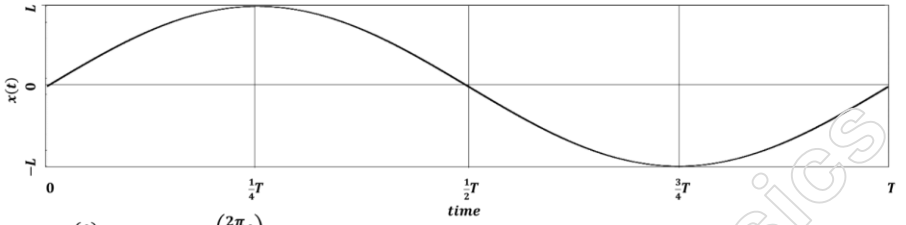
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Representations of SHM

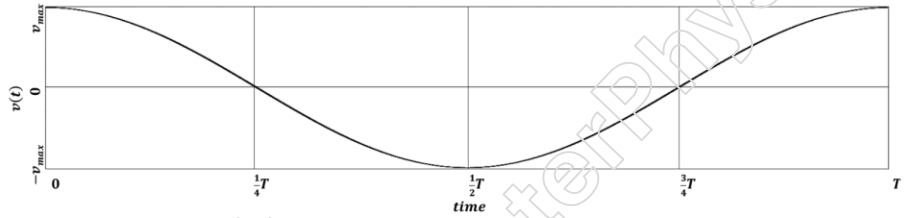
Representations of SHM: Equilibrium Start

Variables as a function of time for 1 period

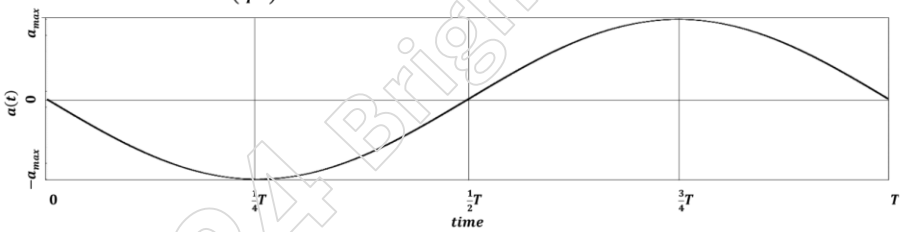
$x(t) = L\sin\left(\frac{2\pi}{T}t\right)$



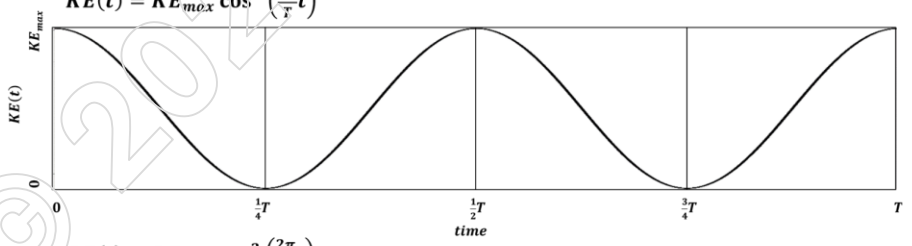
$v(t) = v_{max}\cos\left(\frac{2\pi}{T}t\right)$



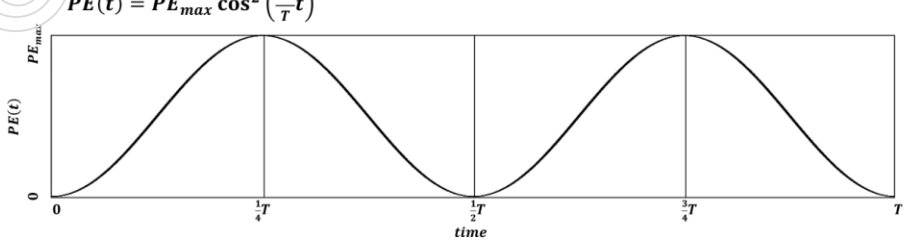
$a(t) = -a_{max}\sin\left(\frac{2\pi}{T}t\right)$



$KE(t) = KE_{max}\cos^2\left(\frac{2\pi}{T}t\right)$



$PE(t) = PE_{max}\cos^2\left(\frac{2\pi}{T}t\right)$



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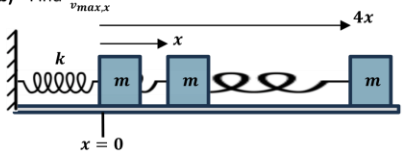
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SHM Springs Four Square

1. A mass $\boldsymbol{m}$ is attached to a spring with force constant $\boldsymbol{k}$. The mass is first pulled a distance $\boldsymbol{x}$ from equilibrium and released, and then pulled a distance $\boldsymbol{4x}$ from the equilibrium position and released.

a) Find $\frac{T_{4x}}{T_x}$

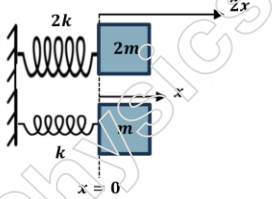
b) Find $\frac{v_{max,4x}}{v_{max,x}}$



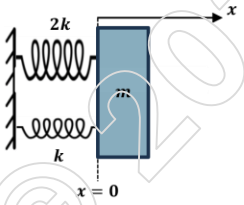
2. A mass $\boldsymbol{m}$ is attached to a spring with force constant $\boldsymbol{k}$ and pulled a distance $\boldsymbol{x}$ from equilibrium and released. A second mass $\boldsymbol{2m}$ is pulled a distance $\boldsymbol{2x}$ from equilibrium and released.

a) Find $\frac{T_{2x}}{T_x}$

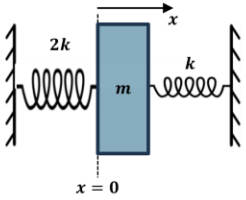
b) Find $\frac{KE_{max,2x}}{KE_{max,x}}$



3. A $\boldsymbol{0.25\ kg}$ mass is attached to two springs that have different spring constants. The mass is pulled $\boldsymbol{15\ cm}$ from equilibrium and released. What is the period of oscillation if $\boldsymbol{k = 200\ \frac{N}{m}}$?



4. A $\boldsymbol{0.25\ kg}$ mass is attached to two springs that have different spring constants. The mass is pulled $\boldsymbol{15\ cm}$ from equilibrium and released. What is the period of oscillation if $\boldsymbol{k = 200\ \frac{N}{m}}$?



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SHM
Springs
Four Square

Student Practice
Blank

Gravitational vs Inertial Mass

The AP inevitably will ask a question on the difference between inertial and gravitational mass, typically in multiple choice section of the test. Typically to choose the proper experimental procedure to unambiguously determine either the gravitational or the inertial mass.

Conceptually, the gravitational and inertial mass are different, but experimentally we find that they are numerically equal. If we take a step back and look at $\vec{F}_{net} = m\vec{a}$, the mass seen in this equation is the inertial mass. The gravitational mass is what is seen in the expressions mg and $G\frac{m_1 m_2}{r^2}$. The fact that they are equal is a restatement of the equivalence principle as given by Einstein – we cannot tell the difference between a constant acceleration and being in a constant gravitational field.

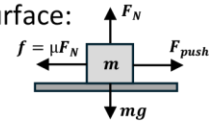
So, given 2nd law equation, if $\vec{a} = 0$ then any mass surviving will be the gravitational mass. Similarly, if $\vec{a} \neq 0$ and there is no gravitational force acting, then any mass will be inertial.

There is one more possibility – the mass has a non-zero acceleration and there is a gravitational force acting. Here it can be ambiguous. But if the gravity is not "driving the motion", than it will still be inertial mass.

Some examples:

Mass moving with constant velocity, $(\vec{a} = 0)$, along horizontal surface:

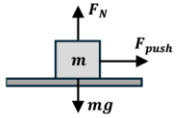
$m = \frac{F_{push}}{\mu g}$ is gravitational mass



Mass accelerating across frictionless horizontal surface:

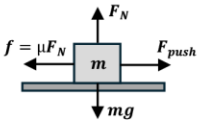
$m = \frac{F_{push}}{a}$ is inertial mass

This push force can be a spring, kx.



Mass accelerating across horizontal surface with friction:

$m = \frac{F_{push}}{a+\mu g}$ is ambiguous because there is a mass associated with the acceleration (inertial mass) and with friction via mg (gravitational mass)



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Gravitational vs Inertial Mass

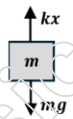
Gravitational vs Inertial Mass

More interesting example occur with springs and pendula, which is why these notes come in this unit rather than in the forces unit.

Some examples:

Mass hanging from a rope (T) or a spring (kx), ($\vec{a} = 0$):

$m = \frac{kx}{g}$ is gravitational mass



Mass oscillating up and down on a vertical spring, ($\vec{a} \neq 0$):

$kx - mg = ma$ looks like it is ambiguous.

However, if we define $x' \equiv x - \frac{mg}{k}$ then we have

$kx' = ma$ and so $m = \frac{kx'}{a}$, an inertial mass. [Think Calculus, the acceleration is the second derivative of the position – the term $\frac{mg}{k}$ is a constant , so a is unaffected.]

With the mass suspended by the spring, the equilibrium position of the spring shifts downwards by $\frac{mg}{k}$ compared to a horizontal arrangement, as in the middle example on the previous page. Gravity does not affect the motion except shifting the equilibrium position – the motion is driven by the spring, not by gravity.

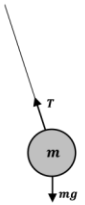
This is ultimately the reason why the period for a spring does not depend on the gravitational field: $T = 2\pi\sqrt{\frac{m}{k}}$. This mass is the inertial mass of the oscillating mass regardless of the geometry.

Mass attached to s string swinging from the ceiling – simple pendulum:

$-mgsin(\theta) = ma$

looks like it is ambiguous, but the mass cancels out

This is ultimately the reason why the period for a simple pendulum does not depend on the mass: $T = 2\pi\sqrt{\frac{L}{g}}$.



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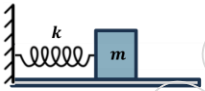
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SHM Four Square

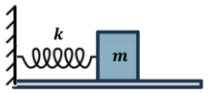
1. A pendulum that has a period $T_E = 1.25\ s$ on the surface of the Earth is transported to the surface of the moon. What is its period T_M on the surface of the moon?

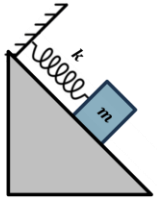
2. What is the length of a simple pendulum that has a frequency $f = 0.25\ Hz$?

3. The spring system shown below has a period $T_E = 1.25\ s$ on the surface of the Earth and is transported to the surface of the moon. What is its period T_M on the surface of the moon?



4. The spring system shown below left has a period $T_E = 1.25\ s$. What will be the new period if placed on a $45°$ inclined plane as shown below right?





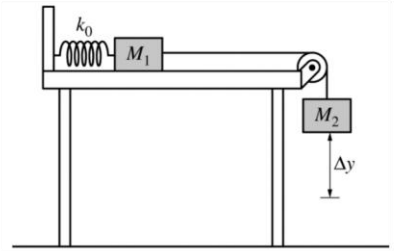
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SHM
Four Square

Student Practice
Blank

AP 2022 #1 Oscillations



Two blocks are connected by a string that passes over a pulley, as shown above. Block 1 is on a horizontal surface and is attached to a spring that is at its unstretched length. Frictional forces are negligible in the pulley's axle and between the block and the surface. Block 2 is released from rest and moves downward before momentarily coming to rest.

k_0 is the spring constant of the spring.
M_1 is the mass of block 1.
M_2 is the mass of block 2.
Δy is the distance block 2 moves before momentarily coming to rest.

(a)

i. Block 2 starts from rest and speeds up, then it slows down and momentarily comes to rest at a position below its initial position. In terms of only the forces directly exerted on block 2, explain why block 2 initially speeds up and explain why it slows down to a momentary stop.

ii. Derive an expression for the distance Δy that block 2 travels before momentarily coming to rest. Express your answer in terms of k_0, M_1, M_2, and physical constants, as appropriate.

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AP 2022 #1 Oscillations

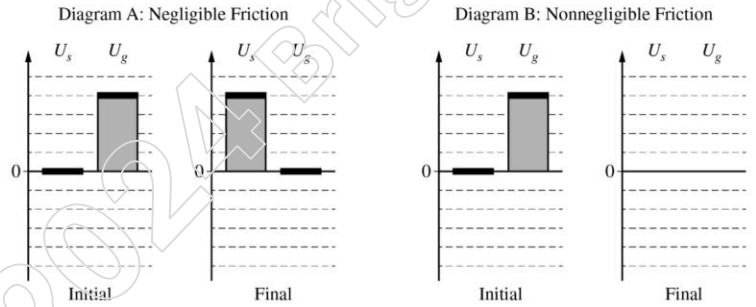
(b) Indicate whether the total mechanical energy of the blocks-spring-Earth system changes as block 2 moves downward.

___ Changes ___ Does not change

Briefly explain your reasoning.

Consider the system that includes the spring, Earth, both blocks, and the string, but not the surface. Let the initial state be when the blocks are at rest just before they start moving, and let the final state be when the blocks first come momentarily to rest. Diagram A at left below is a bar chart that represents the energies in the scenario where there is negligible friction between block 1 and the surface.

The shaded-in bars in the energy bar charts represent the potential energy of the spring and the gravitational potential energy of the blocks-Earth system, U_s and U_g, respectively, in the initial and final states. Positive energy values are above the zero-point line ("0") and negative energy values are below the zero-point line.



(c) Complete diagram B (at right above) for the scenario in which friction is nonnegligible. The energies for the initial state are already provided. Shade in the energies in the final state using the same scale as in diagram A.

- Shaded regions should start at the solid line representing the zero-point line.
- Represent any energy that is equal to zero with a distinct line on the zero-point line.

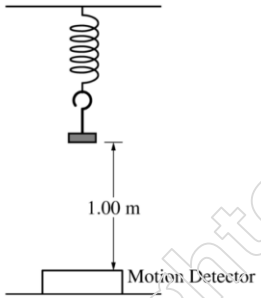
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AP 2022 #5 Oscillations



A spring of unknown spring constant k_0 is attached to a ceiling. A lightweight hanger is attached to the lower end of the spring, and a motion detector is placed on the floor facing upward directly under the hanger, as shown in the figure above. The bottom of the hanger is 1.00 m above the motion detector.

A 0.50 kg object is placed on the hanger and allowed to come to rest at the equilibrium position. The spring is then stretched downward a distance d_0 from equilibrium and released at time $t = 0$. The motion detector records the height of the bottom of the hanger as a function of time. The output from the motion detector is shown in the graph on the following page.

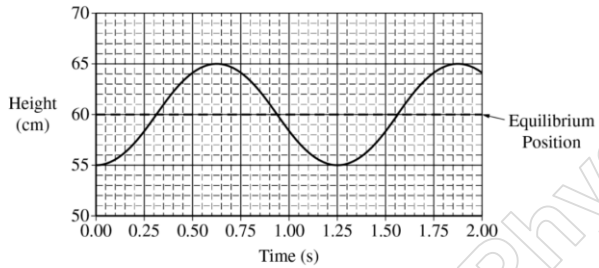
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(a) Using the information given and information taken from the graph, calculate the spring constant.

(b) At time 0.75 s, the object-spring-Earth system has a total kinetic energy K_0 and a total potential energy U_0. At 1.13 s, the object-spring-Earth system again has a total kinetic energy K_0 and a total potential energy U_0.

i. Explain how a feature of the graph indicates that the total kinetic energy of the system is the same at these two times.

ii. Briefly explain why the total potential energy of the system is the same at these two times.

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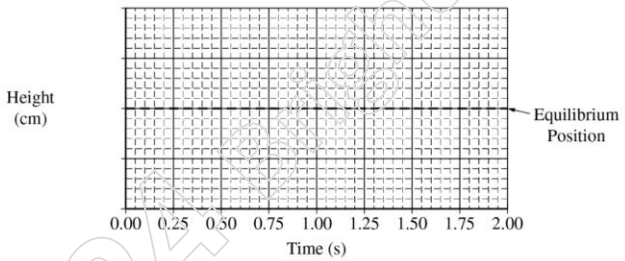
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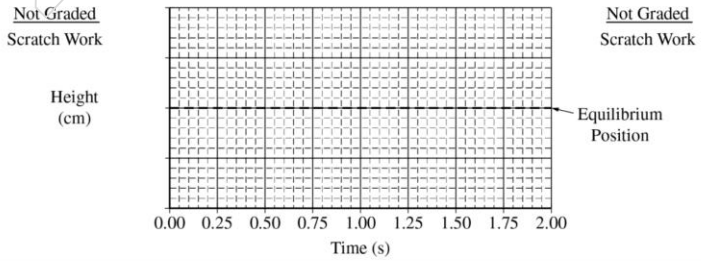
(c) The experiment is repeated with a spring of spring constant $4k_0$ and that has the same length as the original spring. The 0.50 kg object is hung from the new spring and allowed to come to rest at a new equilibrium position.

i. Determine the new equilibrium position above the motion detector.

ii. The object is again pulled down the same distance d_0 from the equilibrium position and released. On the following graph, draw a curve representing the motion of the object after it is released. Label the vertical axis with an appropriate numerical scale. A grid for scratch (practice) work is also provided.



The following graph is provided for scratch work only and will not be graded.



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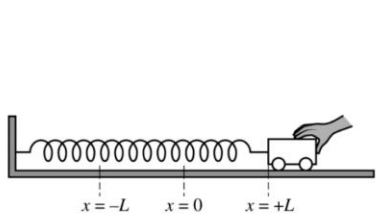


Figure 1

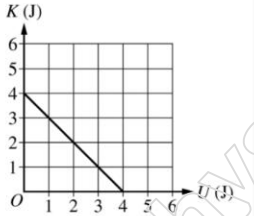


Figure 2

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions $x = +L$ and $x = -L$. Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy K versus potential energy U shown in Figure 2, the values for the x-intercept and y-intercept are the same. Briefly explain why this is true, using physics principles.

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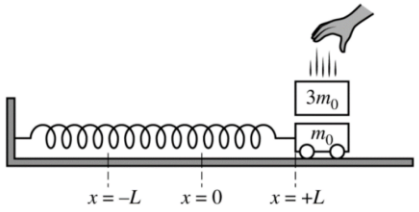


Figure 3

When the cart is at $+L$ and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between $-L$ and $+L$. The masses of the cart and the block are m_0 and $3m_0$, respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is f_1. The frequency of oscillation after the block is dropped onto the cart is f_2. Calculate the numerical value of the ratio $\frac{f_2}{f_1}$.

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(c) The dashed line in Figure 4 shows the kinetic energy K versus potential energy U of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

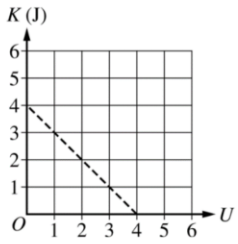


Figure 4

i. Briefly explain why the two graphs must be the same, using physics principles.

ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

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