Momentum

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Momentum
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Introduction to Momentum

Introduction to Momentum

The momentum of an object is defined to be $\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$; both the mass and the velocity must be considered. This means that lighter objects can have more momentum that heavier object if the lighter object is moving quickly enough, and slower objects can have more momentum than faster objects if the slower object is heavy enough. So, for example, a parked car has less momentum than a flying mosquito, and a $15\ mph$ bicycle has less momentum than a $1\ mph$ freight train.

Depending on the situation, we might want to consider the momentum of an individual object, or we might want to consider the momentum of an entire system that contains multiple objects. The total momentum of a system is simply the vector sum of all the individual masses that comprise the system. So for a system consisting of two masses, m_1 and m_1, with velocities $\vec{v}_1$ and $\vec{v}_2$, we can write $\vec{P}_{total} = \vec{p}_1 + \vec{p}_2$, where $\vec{p}_1 = m_1\vec{v}_1$ and $\vec{p}_2 = m_2\vec{v}_2$.

There is no standard convention for when to use upper- or lower-case letters to represent the momentum. One that makes logical sense to me is using a lower case letter for individual objects, and an upper case letter for a system that consists of more than one object, as in the previous paragraph above.

Momentum is often referred to as the ***quantity of motion***, which is the terminology used by Newton who did not use mass and velocity separately. It was an atomist point of view - think infinitesimals! If you are sliding down a ramp, for instance, every molecule of you has the same quantity of motion (assuming each molecule is the same) and so your total quantity of motion is the vector sum of all your constituent molecules. The separation into mass and velocity came later.

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Introduction to Momentum

Introduction to Momentum

Since $\vec{v}$ is a vector, then $\vec{p}$ is also a vector. As such, the frame of reference chosen to determine the velocity $\vec{v}$, and therefore $\vec{p}$, will make a difference. For instance, a passenger inside a moving car will have a momentum of 0 with respect to the car, but not zero with respect to the ground.

In addition, for a given reference frame, the x- and y-components of the momentum for each object must be considered separately. For example, two objects with the same mass m and speed v moving in the same direction will have a momentum of magnitude $2mv$; if moving directly towards each other will have a momentum of 0; if moving in perpendicular directions will have a momentum of magnitude $\sqrt{2}mv$.

If an object is moving, then its momentum cannot be zero. This is true for a single object, but not necessarily for a system. If a system consists of more than one object, the total momentum of the system can be zero even though the individual momenta are not.

There is no special unit for momentum – in standard units, it is $kg \cdot \frac{m}{s}$, or $N \cdot s$ from the Impulse-Momentum theorem.

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Momentum Intro Worksheet

1. Use the information in the table below to calculate the momentum of each object.

Object	Mass (kg)	Velocity (m/s)	Momentum(kg m/s)
bird	0.04	19.	
football player	100.	10.	
skier	60.	20.	
bullet	0.004	600.	
frog	0.9	12.	
meteorite	0.1	1,000.	
aircraft carrier	91,400,000.	0.	
baseball	0.14	30.	
rocket	36,000.	1,800.	
wagon	2.	3.	
satellite	3,000.	8,000.	

2. List these objects in order of increasing momentum: _____

_____

_____

3. What must be the speed of a bird to have the same momentum as the bullet above?

4. What must be the speed of a bullet to have the same momentum as the skier?

5. What must be the mass of a truck with a momentum of 150,000. kg m/s if it traveling with a speed of 30. m/s?

6. A 500 kg motorcycle is moving at 140 km/hr South.
   a) What is the motorcycle's momentum?

   b) At what velocity is the momentum of a 1800 kg car equal to that of the motorcycle?

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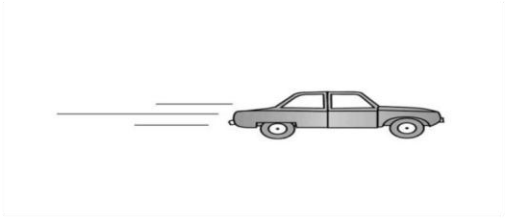
Momentum
Intro
Worksheet

Student Practice
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Impulse Momentum Theorem

Impulse-Momentum

Suppose we consider a car moving with a constant velocity down the road:



Question: **How can we stop the car?**

Answer: **Apply a Force**

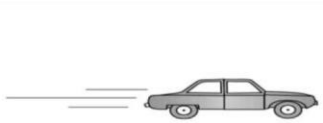
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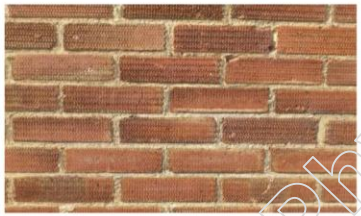
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Impulse Momentum Theorem

Impulse-Momentum

We can apply the force in two different ways:





We can run it into a brick wall

_____

Apply a Large Force





We can run it into a hay pile

_____

Apply a Small Force

In both cases, the car had the same initial momentum.

In both cases, the car had the same final momentum – zero.

In both cases, the Change in the Momentum, ΔP, is the same!

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Impulse Momentum Theorem

Impulse-Momentum

Let's think about the kinematic variables for a moment:

In both cases, v_i and v_f are the same. And we know that the acceleration a is different, because F is different for the two cases.

So that leaves Δt and Δx – both of which are different for the two cases.

- For the large force, the car stops quickly and does not travel far.
- For the small force, the car takes longer to stop and travels farther.

What is $F\Delta x$? This is work, and leads to the Work-Energy Theorem. [Keep in mind that $KE = \frac{p^2}{2m}$, so it is not proportional to p.] So that leaves $F\Delta t$.

Impulse Momentum Theorem:

$$\vec{F}\Delta t = \Delta\vec{p}$$

Note:
The impulse, $\vec{F}\Delta t$, is sometimes given the symbol $\vec{J}$.
Personally, I am not a fan – why introduce another symbol?

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Impulse Momentum Theorem

Impulse Momentum Theorem:

$$\vec{F}_{net}\Delta t = \Delta\vec{p}$$

Let's look at this more closely. Since $p = mv$:

$$\vec{F}_{net}\Delta t = \Delta(m\vec{v})$$
$$= m\Delta(\vec{v}) + \vec{v}\Delta(m)$$

Dividing by Δt:

$$\vec{F}_{net} = m\frac{\Delta\vec{v}}{\Delta t} + \vec{v}\frac{\Delta m}{\Delta t}$$

$$\vec{F}_{net} = m\vec{a} + \vec{v}\frac{\Delta m}{\Delta t}$$

So, Newton's 2nd Law is only an approximation.

The last term involves the rate at which the mass of the object changes.
Does a sliding rock change mass? – No.
A car driving down the street? – Yes (very slowly as the gas is burned)
So we do not typically worry about it.

It does, however, become very important in rocketry. When you see a rocket launch into space, roughly 90% of the weight of the rocket is fuel, which is consumed in just a few minutes. That last term becomes very important.

For a practicing Physicist, the fundamental definition of force is not

$$\vec{F}_{net} = m\vec{a},$$

but rather

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

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Impulse Momentum Theorem

$$F_{net}\Delta t = \Delta p$$

Relationship to Kinematics:

Let's look at this equation, but without the vector notation – so one-dimensional motion. This equation was Newton's starting point, not kinematics. The results for kinematics can be obtained by working backwards from here.

Remember, it was assumed in kinematics that we were dealing with point particles, and therefore non-divisible. So therefore, the mass will not change, and it will cancel out of the equation, leaving $a_{ave}\Delta t = \Delta v$, and then by analogy, $v_{ave}\Delta t = \Delta x$, and recovering the kinematic equations I and IV of the kinematics unit if we assume a is constant.

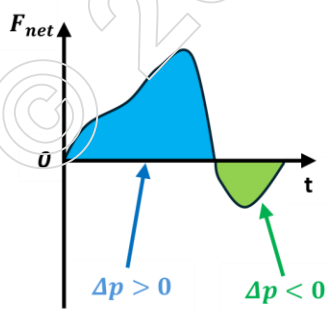
The Impulse-Momentum Theorem is identical to kinematics in the case of indivisible point masses undergoing constant acceleration. For masses that have size, we need to bring in rotational motion for both kinematics and a rotational version of the impulse-momentum equation. For masses that can change, and/or an acceleration that is not constant, we need the full power of the Impulse-Momentum Theorem, and the need to look towards infinitesimals and ultimately Calculus to accommodate both changing mass and acceleration.

Graphical:

Just as in kinematics, we have this same mathematical structure of the change in two variables in the governing equation, in this case Δt and Δp. And just like in kinematics, we can look at ways to handle, in this case, graphs of F or p vs t.

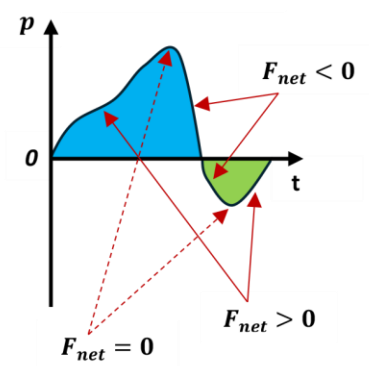
$F_{net}\Delta t = \Delta p$

Look at area



$F_{net} = \frac{\Delta p}{\Delta t}$

Look at slope



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Impulse Momentum Worksheet

1. During his last hockey game, Wayne Gretzky made a slap shot that, fortunately for the opposing team, missed. Suppose that he exerted a constant force of $30.0\ N$ on the hockey puck for $0.16\ s$. What is the magnitude of the impulse given to the puck?

2. Your little cousin has a mass of $35.6\ kg$, and she has a skateboard that has a mass of $1.2\ kg$. What is the combined momentum of your cousin and her skateboard if they are moving at $6.2\ \frac{m}{s}$?

3. During his last hockey game, Wayne Gretzky made a slap shot that, fortunately for the opposing team, missed. Suppose that he exerted a constant force of $30.0\ N$ on the hockey puck for $0.16\ s$. What is the change in the momentum of the puck?

4. POTUS strikes a $0.046\ kg$ golf ball with an average force of $272\ N$. The resulting speed of the ball is $62\ \frac{m}{s}$. (Think – what is the initial speed of the ball?) How long was POTUS's club in contact with the ball?

5. A $0.145\ kg$ baseball is pitched at $42\ \frac{m}{s}$. The batter hits the ball and it travels straight back towards the pitcher at $62\ \frac{m}{s}$. Positive direction is from pitcher to batter. What is the change in the momentum of the ball?

6. A $0.145\ kg$ baseball is pitched at $42\ \frac{m}{s}$. The batter hits the ball and it travels straight back toward the pitcher at $62\ \frac{m}{s}$. Positive direction is from pitcher to batter. If the bat and ball are in contact for $4.6 \times 10^{-4}\ s$, what is the average force exerted on the batted ball?

7. During his last hockey game, Wayne Gretzky made a slap shot that unfortunately missed. Suppose that he exerted a constant force of $30.0\ N$ on the hockey puck for $0.16\ s$. The mass of a hockey puck is $0.16\ kg$. What is the speed of the puck if it is initially at rest before the shot?

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Impulse Momentum Worksheet

Student Practice Blank

Impulse Momentum Questions

$$F_{ave}\Delta t = \Delta p$$

Eggs-aptitude!

Two identical raw organic grade A large eggs are dropped from 2.0 meters above the ground. Egg A lands on a pillow and survives, perhaps one day to hatch into a Perdue Oven Stuffer Roaster Chicken. Egg B lands on concrete and breaks and destined to become a stain on the bottom on someone's Nikes.

The answer to each of the following questions is A (egg A – pillow), B (egg B – concrete), or C (they are the same).

1____ Which egg has the larger speed after it falls the 2.0 meters?

2____ Which egg experiences the greater change in momentum, ΔP?

3____ Which egg experiences the greater average impact force?

4____ Which egg experiences the greater impact time?

5____ Which egg experiences the greater impulse?

Before there were airbags, there were seatbelts!

In the event of a sudden deceleration, the seat belt system in a car quickly, but not instantaneously, locks in place.

6. Explain fully why it is important that the system does not lock instantaneously. (That is, why is it important that the belts give a little?)

7. Ponder this. The belt in a seat belt system is about 2 inches wide. If you were a car company, the thinner you make the belt, the less money it cost to produce the car. Why not, then, make the belt thinner? What would happen during a crash if the belt were, say, only ¼ inch wide, rather than 2 inches?

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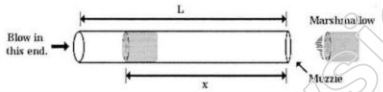
Impulse Momentum Questions

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Marshmallow Launcher Lab

A group of students are conducting an experiment to investigate the factors that impact the effectiveness of a marshmallow launcher. They construct the launcher using hollow PVC pipe. The pipe is cut to varying lengths and loaded with a marshmallow. Blowing in one end of the pipe launches the marshmallow out the other end. The students are using pipes of varying length (**L**) and placing the marshmallow varying distances from the muzzle of the launcher (**x**). These two variables are shown in **Figure 1**.

Figure 1

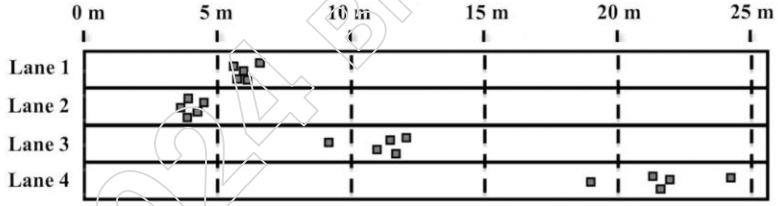


In their experiment, students set up lanes on a hallway and mark distances across the lanes at 5 meter intervals. They shoot the marshmallow down each lane from the sane starting position. They repeat the procedure five times within each lane using a unique length of pipe and starting position of the marshmallow within the pipe. **Table 1** show the values of **L** and **x** that are used in each lane. The locations where the marshmallows land are shown in **Figure 2**.

Table 1

Lane	L	x
1	10 cm	2 cm
2	20 cm	2 cm
3	15 cm	5 cm
4	15 cm	13 cm

Figure 2: Landing Location of Marshmallows



Based on your understanding of the Impulse-Momentum Theorem, answer the following questions below about the above experiment.

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Marshmallow Launcher Lab page 1

Student Practice Blank

Questions:

1. What is the estimated distance traveled through the air by a marshmallow that is launched from a 15 cm long launcher when it is placed 5 cm from the muzzle?
   - **a)** 4 meters
   - **b)** 8 meters
   - **c)** 11 meters
   - **d)** 22 meters

2. Which two lanes best illustrate the effect of the distance of the marshmallow from the muzzle on the landing location of the marshmallow?
   - **a)** Lanes 1 and 2
   - **b)** Lanes 1 and 3
   - **c)** Lanes 2 and 4
   - **d)** Lanes 3 and 4

3. Trials 3 and 4 were conducted in Lanes 3 and 4. What variable was the independent variable in these two trials?
   - **a)** The mass of the marshmallow.
   - **b)** The length of the launcher from which the marshmallow was launched.
   - **c)** The distance that the marshmallow is located from the muzzle of the launcher.
   - **d)** The distance that the marshmallow traveled through the air before landing on the ground.

4. Which of the following would **NOT** be an important variable to control over the course of this study?
   - **a)** The mass of the launcher.
   - **b)** The mass of the marshmallow.
   - **c)** The person who blows on the launcher.
   - **d)** The strength with which the person blows on the open end.

5. Which of the following precautions would not be likely to affect the reliability of this experiment?
   - **a)** Conduct all experiments in the same lane but perform each trail from different heights.
   - **b)** Paint the outside of each launcher a different color so that they can be distinguished from each other.
   - **c)** Conduct all experiments in the same lane but use a different launch angle for each combination of **x** and **L**.
   - **d)** Assign a different student in each lab group a different lane so that each student has a chance to launch the marshmallows.

6. Based on the data presented above, how would the variables L and x need to be manipulated in order to have the longest launch distance?
   - **a)** Both the **L** and **x** should be as long as possible.
   - **b)** Both the **L** and **x** should be as short as possible.
   - **c)** The **L** should be as long as possible and the **x** should be as short as possible.
   - **d)** The **L** should be as short as possible and the **x** should be as long as possible.

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Marshmallow Launcher Lab page 2

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7. Suppose that the results of this study are being used to improve the design of a new NERF gun known as the SuperShot. The gun uses a blast of air to launch a NERF dart through a muzzle. The design team must decide on how long the barrel of the gun should be and where the dart should be positioned in the barrel once it is loaded. Which of the following decisions would improve the SuperShot the most?

a) The barrel of the Supershot should be very short and the dart should be initially located near the muzzle opening of the barrel.

b) The barrel of the Supershot should be very long and the dart should be initially located near the muzzle opening of the barrel.

c) The barrel of the Supershot should be very long and the dart should be initially located as far as possible from the muzzle opening of the barrel.

d) The barrel of the Supershot should be very short and the dart should be initially located as far as possible from the muzzle opening of the barrel.

8. Consider the trends found in the data presented above. Which of the following combinations of tube length (**L**) and distance between marshmallow and the muzzle (**x**) would lead to the shortest distance traveled by a launched marshmallow?

a) **L** = 10 cm, **x** = 8 cm

b) **L** = 10 cm, **x** = 2 cm

c) **L** = 20 cm, **x** = 2 cm

d) **L** = 20 cm, **x** = 18 cm

9. Consider the trends in the data presented above. What would be a reasonable prediction of the landing location for a marshmallow launched by a 20 cm tube when placed 19 cm from the muzzle?

a) Between 4 and 8 meters.

b) Between 8 and 11 meters.

c) Between 11 and 22 meters.

d) More than 22 meters.

10. Which one of the following theories best explains the results obtained in Lanes 3 and 4?

a) More massive marshmallows have a greater resistance to the blowing force and thus leave the launcher at slower speeds.

b) Marshmallows that travel the furthest distance through the pipe will encounter the greatest friction and leave the muzzle with the lowest speeds.

c) The longer the length of the pipe through which the marshmallow is blown, the greater the force that is exerted on the marshmallow and the greater the speed.

d) The longer the time the blowing force acts on the marshmallow, the greater the momentum that the marshmallow will obtain when being launched.

11. The dependent variable in this experiment is _____.

a) The length of the pipe.

b) The mass of the marshmallow.

c) The landing location of the marshmallow.

d) The distance of the marshmallow to the muzzle of the pipe.

e) Two of the above.

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Marshmallow Launcher Lab page 3

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Conservation of Momentum

Conservation of Momentum

A quantity is conserved if it has not changed because of some process that takes place. Technically the requirement is that it is the same after as before the event – it might change during, however. This allows, for example, a catalyst that is consumed and then regenerated as part of a catalytic reaction to be considered as conserved.

For momentum to be conserved during some process, then $\boldsymbol{\Delta P = 0}$. So starting from the Impulse Momentum Theorem, $\boldsymbol{F_{net}\Delta t = \Delta P}$, momentum is conserved if, and only if, $\boldsymbol{F_{net}\Delta t = 0.}$

For this to be true, either:

$$\boldsymbol{F_{net} = 0} \quad \text{or} \quad \boldsymbol{\Delta t = 0}$$

If $\boldsymbol{\Delta t = 0}$ then there is no event, no process, no problem!

So, momentum is conserved if, and only if, $\boldsymbol{F_{net} = 0}$ during the process. Keep in mind that, since forces and momenta are vectors, there is the possibility of momentum being conserved in one direction but not another. For instance, in projectile motion, assuming no air friction, the only force acting is vertical. The momentum is therefore conserved horizontally, which means that $v_x = constant$ in projectile motion events.

It was mentioned in the forces unit that the direction of $\vec{\boldsymbol{F}}_{\boldsymbol{net}}$ represents a preferred coordinate direction. If it is a non-zero constant, then the momentum in any direction perpendicular to that direction is conserved. In the direction of $\vec{\boldsymbol{F}}_{\boldsymbol{net}}$, $\boldsymbol{a \neq 0}$ and $\boldsymbol{\Delta P \neq 0}$.

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Conservation of Momentum

For momentum to be conserved during some process, then $\boldsymbol{\Delta P = 0}$. So starting from the Impulse Momentum Theorem, $\boldsymbol{F_{net}\Delta t = \Delta P}$, momentum is conserved if, and only if, $\boldsymbol{F_{net}\Delta t = 0}$. For this to be true:

$$\boldsymbol{F_{net} = 0} \quad \text{or} \quad \boldsymbol{\Delta t = 0}$$

If $\boldsymbol{\Delta t = 0}$ then there is no event, no process, no problem! In real life macroscopic collisions however, there are often external friction forces acting on the masses within the system during the collision. If the collision time, Δt, is small then the momentum is approximately true. The issue is that if the external force is a non-conservative force, friction for example, then work is done on or by the system during the collision time. During a car accident, for instance, the actual collision time is only a few tenths of a second, the cars to not travel but a few inches during the actual collision, and so little energy leaves the system. After the collision, the cars can move quite far, and the cars skid to a stop.

There may be internal energy losses, due to friction and deformations **within** the system. This will distinguish between elastic and inelastic collision that we will look after the example conservation of momentum problems below. These energy considerations are separate from the energy losses due to external forces acting on the system that would invalidate the conservation of momentum.

In the following conservation of momentum problems, the masses are on ice, minimizing the external friction acting on the skaters as they collide.

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Conservation of Momentum Videos

Conservation of Momentum Videos

The links to two videos that I made during the pandemic are offered below:

Conservation of Momentum – collision of two carts:

https://youtu.be/-tyait29Pdc



This includes a discussion of what a system is and why it is important to keep the system in mind when we analyze the motion that we see. Both conservation and impulse-momentum are discussed.

Rock Your World – You too can move the world:

https://youtu.be/NxK35fUTz7w



This includes moving the Earth horizontally by jumping against a wall, and vertically by dropping a set of keys. Both conservation and impulse-momentum are discussed.

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General Procedure for any Conservation of Momentum Problem

Conservation of Momentum

We need a few definitions to be able to make clear and concise statements regarding momentum problems.

System:

- The set of objects under study

Closed system:

- objects cannot enter or leave the system

Isolated system:

- No net external force acting on the system

The momentum of a closed, isolated system is conserved

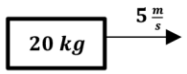
Conservation of Momentum Problem Solving Procedure

1. **Physical sketch**
2. **Momentum diagram**
3. **Equation**
4. **Dress**
5. **Solve**

Physical Sketch and Momentum diagram can often be combined.
Momentum diagram:

For each mass

- draw a box with mass inside
- attached velocity vector



The momentum diagram morphs into the equaion

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Example 1:

Dick and Jane are learning to ice skate. Their masses are $75\ kg$ and $40\ kg$, respectively. With a speed of $3\ \frac{m}{s}$, Dick slides into Jane who is initially at rest, and they move off together. What is their final speed?

Physical Sketch:





Momentum Diagram:



Conservation Equation: $mv + mv = mv$

Dress: $(40)(3) + (75)(0) = (115)(v_f)$

Solve: $v_f = 1.96\ \frac{m}{s}$

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Example 2:

Dick and Jane are learning to ice skate. Their masses are $75\ kg$ and $40\ kg$, respectively. Dick skates at $2\ \frac{m}{s}$ toward Jane and Jane skates $3\ \frac{m}{s}$ toward Dick. They collide and hold on tight. What is their final speed?

Physical Sketch / Momentum Diagram:



Conservation Equation: $mv + mv = mv$

Dress:

$$(75)(2) + (40)(-3) = (115)(v_f)$$

$$(75)(2) - (40)(3) = (115)(v_f)$$

Solve:

$$v_f = 0.26\ \frac{m}{s}$$

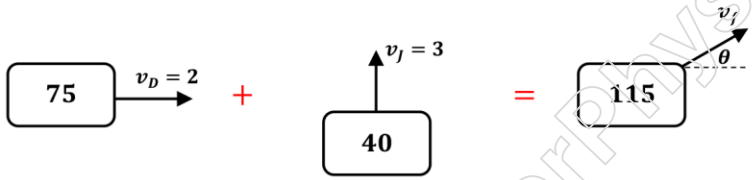
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Example 3:

Dick and Jane are learning to ice skate. Their masses are $75\ kg$ and $40\ kg$, respectively. Dick skates at $2\ \frac{m}{s}\ East$ and Jane skates $3\ \frac{m}{s}\ North$. They collide and hold on tight. What is their final velocity?

Diagram:



Equation: $mv + mv = mv$

Dress:

$$X: \quad (75)(2) + 0 = (115)(v_f \cos\theta) = 115v_{fx}$$

$$Y: \quad 0 - (40)(3) = (115)(v_f \sin\theta) = 115v_{fy}$$

Momentum is conserved in $x-$ and $y-$direction separately. So there are two equations to solve simultaneously.

Solve: $v_f = 1.67\ \frac{m}{s}\ @\ 38.7°\ North\ of\ East$

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Conservation of Momentum Worksheet

1. A 95 kg fullback, running at 8.2 m/s collides in midair with a 128 kg defensive tackle moving in the opposite direction. Both players end up with zero speed. How fast was the defensive tackle moving originally?

2. Marble C, with mass 5g, moves at a speed of 20.0 cm/s. It collides with a second marble, D, with a mass of 10 g, moving at 10 cm/s in the same direction. After the collision, marble C continues with a speed of 8.0 cm/s in the same direction. What is the speed of marble D after the collision?

3. Two lab carts are pushed together with a spring mechanism compressed between them. Upon release, the 5 kg cart repels one way with a speed of 0.12 m/s, while the 2.0 kg cart moves in the opposite direction. What is the speed of the 2.0 kg cart?

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Conservation of Momentum Worksheet page 1

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Conservation of Momentum Worksheet

4. A 50.0 g projectile is launched with a horizontal velocity of 647 m/s from a 4.65 kg launcher moving in the same direction at 2.00 m/s. What is the launcher's velocity after the launch?

5. A 12.0 g rubber bullet travels at a velocity of 150 m/s , hits a stationary 8.5 kg concrete block resting on a frictionless surface, and ricochets in the opposite direction with a velocity of -1.0×10^2 m/s. How fast will the concrete block be moving?

6. Kofi, with a mass of 42.00 kg, is riding a skateboard with a mass of 2.00 kg and traveling at 1.20 m/s. Kofi jumps off and the skateboard stops dead in its tracks. In what direction and with what speed did he jump?

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Conservation of Momentum Worksheet page 2

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Turkey Challenge

Chef Boy-R-Dee has developed a new and improved stuffing recipe. He claims that it is better tasting than Stove Top, but will in fact decrease the cooking time for a large stuffed turkey. And prevents injury by removing the need for a large sharp knife to carve the turkey once cooked. The secret ingredient – Trinitrotoluene!

A $30\ lb$ turkey is place in a 350° oven. After 2.0 hours, the TNT explodes, carving the turkey into several large pieces. A $10.5\ lb$ breast moves East at $5\ \frac{m}{s}$; a $0.75\ lb$ wing flies (get it?) at $10\ \frac{m}{s}\ 53°\ N\ of\ W$; a $2.5\ lb$ drumstick travels at $0.2\ \frac{m}{s}$ at $30°\ S\ of\ E$; a $0.5\ lb$ thigh remains motionless. What is the velocity $\vec{v}$ of the remaining part of the

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Conservation of Momentum Worksheet

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Collision Problems

Collisions

The two pillars in experimental Physics are the two conservation laws Conservation of Momentum and Conservation of Energy. In this unit we have looked at momentum. At this point in the course, students may or may not have seen energy depending on the sequencing. But they have seen enough in their Chemistry and Biology classes for what is needed here. Both of these conservation laws are relevant in collision problems.

When a system consists of two objects colliding, assuming no outside forces interfering, momentum conservation is valid, as well as the total energy. Mechanical energy may not, however. It depends on whether or not there are any internal non-conservative forces doing work **within** the system. The two more obvious examples being friction (if the objects rub against each other) and permanent deformations (breaking glass, crumpled hood and bumpers, etc. if we think of a car collision).

So collisions can be categorized according to what happens to the mechanical energy, specifically the kinetic energy ($\frac{1}{2}mv^2$), during the collision. [Generally, we consider collisions here between objects that interact through contact forces – they touch. If we considered a gravitational or electrostatic interaction between objects, for example, then they don't ever not interact, and would substitute mechanical energy for kinetic energy in what follows.]

Collisions are categorized into elastic or inelastic collisions, depending on whether or not the total kinetic energy of the system changes or not as a result of the collision.

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Collision Problems

Collisions

Let's introduce a few definitions:

Kinetic Energy:

- Energy of motion: $\boldsymbol{KE} \equiv \frac{1}{2}mv^2$.

Elastic Collision:

- collision where the **total kinetic energy** before and after the collision **are** equal
- sometimes wrongly referred to as perfectly elastic
    - this would imply that there is an imperfectly elastic collision, which can only mean that KE is not quite conserved, which by definition means it is inelastic!

Inelastic Collision:

- collision where the **total kinetic energy** before and after the collision **are not** equal

Totally Inelastic Collision:

- collision where the objects that collide stick together
- there is only one object after the collision (or before, if the process is an explosion)
- as the name implies, it is necessarily an inelastic collision
- sometimes referred to a **perfectly inelastic collision**

In all three cases, the total momentum is conserved.

What happens to the energy? Deformations that are not reversible occur; something heats up; sound is emitted; anything that removes kinetic energy from the system permanently. [It can happen that energy is added to the system instead: for example, gas tank explodes during the collision; fire-cracker explodes.]

This non-reversibility is really at the heart of what it means to be elastic. To bounce is not sufficient for being elastic. Drop a superball onto the floor – it does not rise to the original height after bouncing off the ground. It is NOT elastic. Think of the rubber band lab, or bending a paper clip – if kept in the Hooke's Law region, a deformation is reversible.

Elasticity is a measure of how close the object comes to being elastic. An elasticity of 0 means the objects stick together. An elasticity of 100% means that no KE is lost due to the collision – it is elastic.

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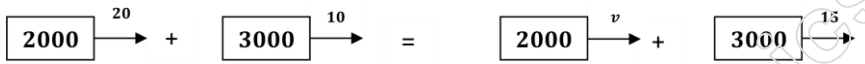
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Elastic/Inelastic Collision Example

A 2000 kg car moving to the right at 20 $\frac{m}{s}$ strikes a 3000 kg truck traveling to the right at 10 $\frac{m}{s}$. After the collision, the truck is moving at 15 $\frac{m}{s}$ after the collision.

a) What will be the velocity of the car after the collision?
b) Was this collision elastic or inelastic?

To answer the first question, we solve the problem using conservation of momentum as before:



$$mv + mv = mv + mv$$

$$(2000)(20) + (3000)(10) = (2000)(v) + (3000)(15)$$

$$40000 + 30000 = 2000\,v + 45000$$

$$70000 = 2000\,v + 45000$$

$$25000 = 2000\,v$$

$$v = 12.5\,\frac{m}{s}$$

To determine if this collision is elastic or not, we need to determine the kinetic energy before and after the collision.

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 \;?=?\; \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2$$

$$\tfrac{1}{2}(2000)(20)^2 + \tfrac{1}{2}(3000)(10)^2 \;?=?\; \tfrac{1}{2}(2000)(12.5)^2 + \tfrac{1}{2}(3000)(15)^2$$

$$400000 + 150000 \;?=?\; 156250 + 337500$$

$$550000\,J \neq 493750\,J$$

This collision is **inelastic** because the total kinetic energy before the collision is not equal to the kinetic energy after the collision.

In solving general momentum problems where the collision is explicitly elastic necessarily results in a quadratic equation since momentum is a linear equation while energy is quadratic.

Typically, one solution will be the situation before the collision, and the other for the situation after the collision. Examples of this issue are in the student practice Collision Problems Worksheet.

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Collision Problems Worksheet

Problems:

1. A 2 kg mass moving at 3 $\frac{m}{s} East$ collides with a 4 kg mass at rest. Afterwards, the 2 kg mass recoils 1 $\frac{m}{s} West$.
   a) What is the velocity of the 4 kg mass after the collision?
   b) Determine if this collision is elastic or inelastic.

2. A 1 kg mass moving at 9 $\frac{m}{s} East$ collides with a 2 kg mass moving $West$ at 3 $\frac{m}{s}$. Afterwards, the 1 kg mass recoils 3 $\frac{m}{s} West$.
   a) What is the velocity of the 2 kg mass after the collision?
   b) Determine if this collision is elastic or inelastic.

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Collision Problems Worksheet page 1

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Collision Problems Worksheet

3. A 5 kg mass moving at 9 $\frac{m}{s}$ $East$ collides with a 10 kg mass moving $West$ at 3 $\frac{m}{s}$. Afterwards, the 5 kg mass recoils 3 $\frac{m}{s}$ $West$.
   a) What is the velocity of the 10 kg mass after the collision?
   b) Determine if this collision is elastic or inelastic.

4. A 5 kg mass moving 2 $\frac{m}{s}$ $East$ collides with a 5 kg mass at rest. If the collision is totally inelastic, what are the velocities of each ball after the collision?

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Collision Problems Worksheet page 2

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Collision Problems Worksheet

5. A 5 kg mass moving $East$ at 2 $\frac{m}{s}$ collides with a 5 kg mass at rest. If the collision is elastic, what are the velocities of each ball after the collision? [Hint, you will have 2 equations to solve – one from conservation of momentum, and one from the fact that kinetic energy before and after the collision is equal.]

6. A 2 kg mass moving $East$ at 5 $\frac{m}{s}$ collides with a 5 kg mass moving $East$ at 2 $\frac{m}{s}$. If the collision is elastic, what are the velocities of each ball after the collision? [Hint, you will have 2 equations to solve – one from conservation of momentum, and one from the fact that kinetic energy before and after the collision is equal.]

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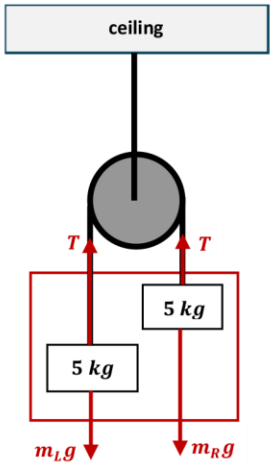
Collision Problems Worksheet page 3

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Center of Mass Notes

Center of Mass

Let's revisit the basic pulley system, with massless frictionless pulley and massless rope:

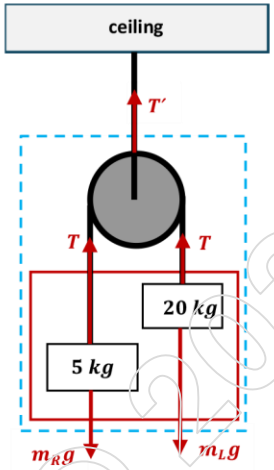


In the forces unit, this was solved by drawing two separate FBD resulting in two simultaneous equations allowing us to determine the tension and acceleration of the rope. In this case $T = 50\ N$ and $a = 0$. So each mass moves with constant speed, one up, one down, and so the center of mass is stationary.

If we consider it as two separate systems, both systems are closed and isolated since the tension was $50\ N$ because the weights are both $50\ N$ so $F_{net} = 0$ on each mass.

As pointed out in the forces unit, $F_{net} = m_{system} a_{system}$. There are two systems, both have mass $5\ kg$ and acceleration of 0. The center of mass of one system moves up, the center of mass of the other system moves down.

Now look at the red system, containing both masses. This one larger system is still closed and isolated. The tensions are both equal to the weight, so $F_{net} = 0$ for the system.



Now let's look at an unbalanced system. The solution to this problem from the forces unit looking at two separate systems is $T = 80\ N$ and $a = 6\ \frac{m}{s^2}\ clockwise$.

Now look at the red system, containing both masses. This is no longer isolated. Applying Newton's 2nd Law with positive upwards:

$$F_{net} = m_{sys} a_{cm}$$

$$T + T - 50 - 200 = (25) a_{cm}$$

$$a_{cm} = -3.6\ \frac{m}{s^2}$$

We could find the tension in the rope connected to the ceiling by looking at the blue FBD. The four tensions T are internal to the blue FBD and cancel. Then

$$F_{net} = m_{sys} a_{cm}$$

$$T' - 50 - 200 = (25)(-3.6)$$

$$T' = 70\ N$$

One has to be careful drawing an FBD with a pulley inside as part of the system. The support structures provide a force to keep the pulley in position (T' in this case). Here the pulley is massless. One might object to the results: if you consider yet another FBD enclosing only the pulley in this case, then you have tension force of $70\ N$ upwards and two tension forces downwards of $80\ N$ each, so there is a net force on the pulley. But the acceleration of the pulley itself is indeterminate mathematically because the pulley is massless, which is consistent with zero!

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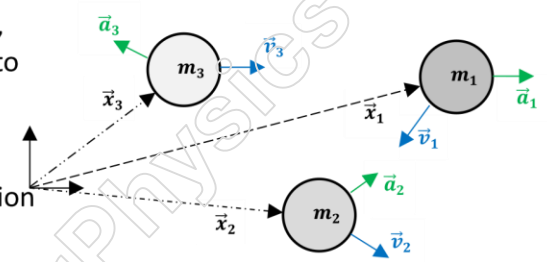
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Center of Mass Notes

Center of Mass

There are a variety of ways to find the $\vec{a}_{cm}$, $\vec{v}_{cm}$, and $\vec{x}_{cm}$ variables for a system of particles. By Assuming a consistent definition of one of them, the others could simply be determined through differentiation and/or integration to find the other two. It is certainly simple enough, but seems like a cheat, and not accessible in the curriculum for AP Physics 1.

But they can be derived from what we already have, except for $\vec{x}_{cm}$ but we shall show here anyway just to have everything in one place.



Let's start with a closed system of n particles where Each particle i has position $\vec{x}_i$, velocity $\vec{v}_i$, acceleration $\vec{a}_i$, and mass m_i with respect to some fixed coordinate system, shown at right for three masses.

$\vec{a}_{cm}$:

We have already seen on the last slide that $\vec{F}_{net} = m_{sys}\vec{a}_{cm}$, but $\vec{F}_{net} = \sum_{i=1}^{n}(\vec{F}_i)$, so $m_{sys}\vec{a}_{cm} = \sum_{i=1}^{n}(m_i\vec{a}_i)$ or $\vec{a}_{cm} = \frac{\sum_{i=1}^{n}(m_i\vec{a}_i)}{m_{sys}}$, agreeing with the result on the last slide. The forces that give rise to each acceleration can be external and internal action-reaction pairs.

$\vec{v}_{cm}$:

If we define $\vec{P}_{net} = m_{sys}\vec{v}_{cm}$, and realize that $\vec{P}_{net} = \sum_{i=1}^{n}(\vec{p}_i)$, then $m_{sys}\vec{v}_{cm} = \sum_{i=1}^{n}(m_i\vec{v}_i)$ or $\vec{v}_{cm} = \frac{\sum_{i=1}^{n}(m_i\vec{v}_i)}{m_{sys}}$.

$\vec{x}_{cm}$:

To derive the position of the center of mass, we can use the idea that the center of mass is the same as the center of gravity. Conceptually, the gravity acting on a distributed mass results in the same motion as if all the mass was concentrated at the center of mass. So we can imagine placing this system in a constant gravitational field, say down in the above image and calculate the torque about the axis through the origin perpendicular to the image shown. The appropriate component of the position to use is then the horizontal component as shown in the image. Then rotate the system to determine the other coordinate direction components. Then $\tau_{net} = (m_{sys}g)\,x_{cm}$ and realize that $\tau_{net} = \sum_{i=1}^{n}[(m_i g)x_i]$, and then for y and z, then $m_{sys}\vec{x}_{cm} = \sum_{i=1}^{n}(m_i\vec{x}_i)$ or $\vec{x}_{cm} = \frac{\sum_{i=1}^{n}(m_i\vec{x}_i)}{m_{sys}}$.

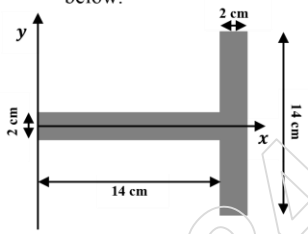
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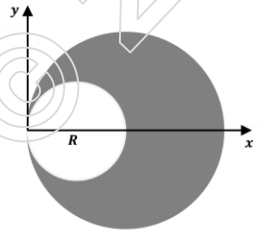
Center of Mass Worksheet

1. The distance between the centers of the carbon ($m_C = 12u$) and oxygen ($m_O = 16u$) atoms in a molecule of CO is $1.13 \times 10^{-10}m$. Find the distance of the center of mass of the CO molecule from the carbon atom.

2. Determine the position of the center of mass of the uniform T-construction bracket shown below.



3. A uniform circular disk of radius $2R$ has a circular hole or radius R as shown. Find the position of the center of mass of the disk with the hole.



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Center
of
Mass
Worksheet

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AP Momentum Examples

AP Momentum Examples

Center of mass problems on the AP exam occur in both MC and FRQ parts of the test. In the MC section, they are usually straight forward. For example, a problem might read: A $5\ kg$ mass slides at $3\ \frac{m}{s}$ on a frictionless surface and strikes a $10\ kg$ mass initially at rest. The collision takes $0.5\ s$ and afterwards they stick together. Even though I have not asked a question, we can immediately draw the following:



Obviously finding the final velocity is simple here – it is $1\ \frac{m}{s}$.

Depending on the question there are only a few things we can do. We can apply the Impulse-Momentum Theorem to either of the two masses [treating them as separate systems] and find the average action-reaction forces $F_{5\to10}$ and $F_{10\to5}$ during the $0.5\ s$ collision. Or, since we already know the final velocity, we can find the velocity of the center of mass (which is $1\ \frac{m}{s}$) before or after the collision [treating the two masses together as a single system].

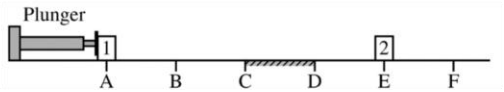
However, if this same problem is embedded into an FRQ, the simplicity of this problem tends to get lost by the rest of the surrounding problem. Students will simply not recognize which system to use.

The next few slides are problems taken from AP FRQs for students to practice AP style question once they have mastered the basic momentum techniques presented in this unit.

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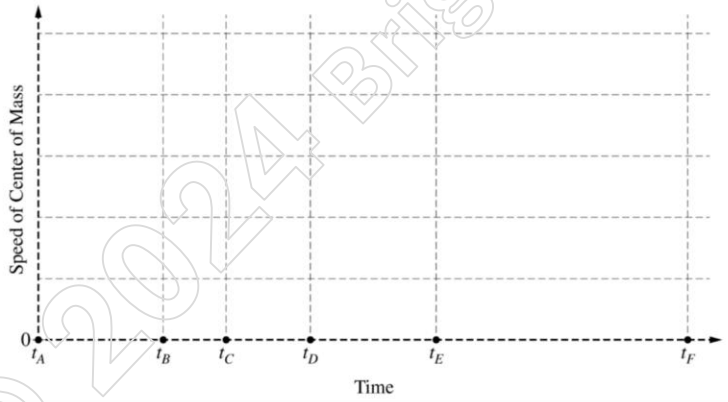
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AP Momentum 2019



Identical blocks 1 and 2 are placed on a horizontal surface at points A and E, respectively, as shown. The surface is frictionless except for the region between points C and D, where the surface is rough. Beginning at time t_A, block 1 is pushed with a constant horizontal force from point A to point B by a mechanical plunger. Upon reaching point B, block 1 loses contact with the plunger and continues moving to the right along the horizontal surface toward block 2. Block 1 collides with and sticks to block 2 at point E, after which the two-block system continues moving across the surface, eventually passing point F.

(a) On the axes below, sketch the speed of the center of mass of the two-block system as a function of time, from time t_A until the blocks pass point F at time t_F. The times at which block 1 reaches points A through F are indicated on the time axis.



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AP Momentum 2019

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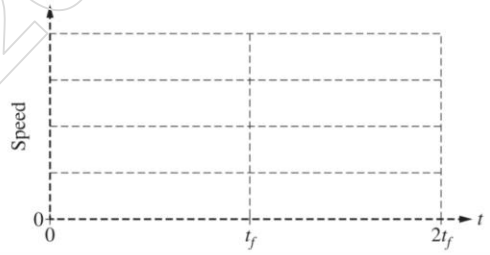
AP Momentum 2021

(a) A student of mass M_S, standing on a smooth surface, uses a stick to push a disk of mass M_D. The student exerts a constant horizontal force of magnitude F_H over the time interval from time $t = 0$ to $t = t_f$ while pushing the disk. Assume there is negligible friction between the disk and the surface.

i. Assuming the disk begins at rest, determine an expression for the final speed v_D of the disk relative to the surface. Express your answer in terms of F_H, t_f, M_S, M_D, and physical constants, as appropriate.

ii. Assume there is negligible friction between the student's shoes and the surface. After time t_f, the student slides with speed v_S. Derive an equation for the ratio v_D / v_S. Express your answer in terms of M_S, M_D, and physical constants, as appropriate.

(b) Assume that the student's mass is greater than that of the disk ($M_S > M_D$). On the grid below, sketch graphs of the speeds of both the student and the disk as functions of time t between $t = 0$ and $t = 2t_f$. Assume that neither the disk nor the student collides with anything after $t = t_f$. On the vertical axis, label v_D and v_S. Label the graphs "S" and "D" for the student and the disk, respectively.



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AP Momentum 2021



Disk Block

(c) The disk is now moving at a constant speed v_1 on the surface toward a block of mass M_B, which is at rest on the surface, as shown above. The disk and block collide head-on and stick together, and the center of mass of the disk-block system moves with speed v_{cm}.

i. Suppose the mass of the disk is much greater than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

ii. Suppose the mass of the disk is much less than the mass of the block. Estimate the velocity of the center of mass of the disk-block system. Explain how you arrived at your prediction without deriving it mathematically.

iii. Now suppose that neither object's mass is much greater than the other but that they are not necessarily equal. Derive an equation for v_{cm}. Express your answer in terms of v_1, M_D, M_B, and physical constants, as appropriate.

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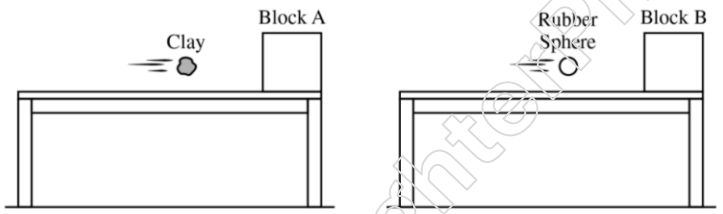
AP Momentum 2021 page 2

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AP Momentum 2022

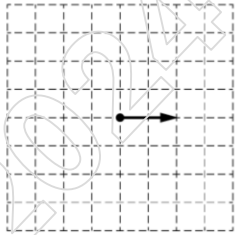
A student has a piece of clay and a rubber sphere, both of the same mass. Both objects are thrown horizontally at the same speed at identical blocks that are at rest at the edge of identical tables, as shown, where friction between the blocks and the table is negligible. After the collisions, both blocks fall to the floor.

In Case A, the clay sticks to Block A after the collision. In Case B, the rubber sphere bounces off of Block B after the collision.

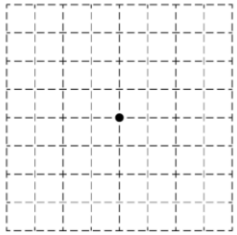


Case A: Clay and Block A
Before Collision

Case B: Rubber Sphere and Block B
Before Collision



Case A: Momentum of
Clay-Block System
Immediately After Collision



Case B: Momentum of
Sphere-Block System
Immediately After Collision

(a) In the figure at left above, the arrow represents the momentum immediately after the collision for the clay-block system in Case A. In the figure at right above, draw an arrow starting on the dot to represent the momentum of the sphere-block system immediately after the collision in Case B. If the momentum is zero, write "zero" next to the dot. The momentum, if it is not zero, must be represented by an arrow starting on, and pointing away from, the dot. The length of the vector, if not zero, should reflect the magnitude of the momentum relative to Case A.

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AP
Momentum
2022
page 1

Student Practice
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AP Momentum 2022

(b) After the clay and Block A collide, Block A lands a horizontal distance d_A from the edge of the table. Does Block B land on the floor at a horizontal distance from the edge of the table that is greater than, less than, or equal to d_A ? In a clear, coherent, paragraph-length response that may also contain equations and/or drawings, explain your reasoning. Neglect any frictional effects due to the table or air resistance.

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2022
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