Forces II

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Forces II

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Introduction to Forces II

Introduction to Forces II

The forces considered in Forces I were limited to constant forces[‡]. This allowed us to focus narrowly on how to use the force procedures developed there to solve force problems without the added distractions of variable forces.

In the Forces II unit, this restriction to constant forces is removed, allowing for forces than can vary not only in magnitude, but in direction as well as in position and time. Gravitational forces; electrostatic forces; magnetic forces; spring forces; forces that result in circular motion; forces that involve pulleys – these forces are handled using the same procedure already presented in Forces I.

As we shall see, the force procedure will remain the same – only the input into that procedure will be refined.

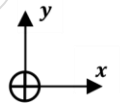
[‡]Air friction force was the only exception. This was done to draw contrast with static and kinetic friction forces between solid objects that are constant.

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Mass on Inclined Planes

A mass on an inclined plane is usually the first place where we (students as well as teachers) typically confront the idea of a coordinate system philosophically. When we think of a coordinate system in two dimensions, most of us immediately envision the coordinate system shown at right. This is more than likely because of our high school math teachers. Using this coordinate system, students are usually quite adept at finding the x and y components of vectors.



But why do we need a coordinate system? The answer is that we are not smart enough yet to deal with vectors as vectors. Newton's 2nd Law, $\vec{F}_{net} = m\vec{a}$, is a vector equation. Vectors are not positive or negative -- the components are. We can deal with components of the vectors and the resultant equation in each of the coordinate directions, but only by referencing a coordinate system. But the coordinate system shown above is only one of an infinite number of choices.

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Mass on Inclined Planes

Looking at Newton's 2nd Law at bit more closely, we notice that the left and right sides of the equation are very different. As a vector, the left-hand side involves perhaps more than one term because it is a sum of vectors – nothing special here. However, as a vector the right-hand side contains a single term. A mass can only undergo a single acceleration, otherwise it will not be a single object for long – it will break apart. There is therefore a "preferred direction" suggested by the vector equation itself – the direction of the acceleration vector!

So, quite generally, the "best choice" for a coordinate system is to choose the direction of the acceleration, if known, as one coordinate direction. The sum of the forces in the direction perpendicular to that choice will necessarily be zero. This simplifies the resulting system of equations that follow from Newton's 2nd Law.

For an inclined plane then, the preferred direction is one where positive is along the ramp and the other perpendicular to the ramp. If the acceleration is zero, then it does not matter. A comparison of the two choices is presented in the next slide.

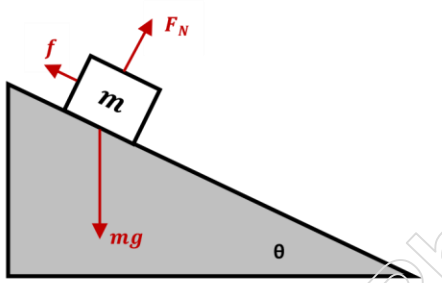
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Coordinate Choice for Inclined Plane Problems

Suppose a mass is placed on an inclined plane with angle θ that has a coefficient of friction of μ.
The FBD for the problem is shown below. The next step is to create the force chart, but using what coordinate system?
There are two obvious coordinate systems to use:

- **standard horizontal/vertical**
- **parallel/perpendicular to ramp**



Let's look at both:





Force	x	y
F_N	$F_N \sin\theta$	$F_N \cos\theta$
f	$-f \cos\theta$	$f \sin\theta$
mg	0	$-mg$
x-equation: $F_N \sin\theta - f\cos\theta = ma\cos\theta$		
y-equation: $F_N \cos\theta + f\sin\theta - mg = ma\sin\theta$		

In this case, if a is not zero then neither equation is equal to zero. And the acceleration itself needs to be decomposed into components.

Force	x'	y'
F_N	0	F_N
f	$-f$	0
mg	$mg\sin\theta$	$-mg\cos\theta$
x'-equation: $mg\sin\theta - f = ma$		
y'-equation: $F_N - mg\cos\theta = 0$		

In this case, the y'-equation will always be equal to zero, and this makes the mathematics easier.

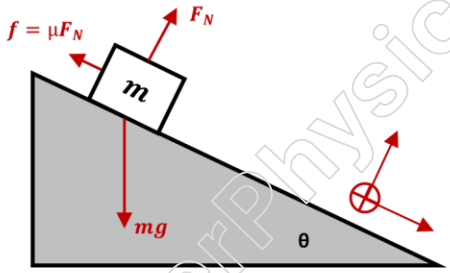
Also, this is consistent with the "best choice" for positive direction being the direction of the acceleration.

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Example Inclined Plane Problem

Suppose a mass is placed on an inclined plane with angle $\theta = 37°$ that has a coefficient of friction of $\mu = 0.15$. The mass is placed on the ramp and released, and the mass then slides down the ramp. Find the acceleration of the mass as it slides down the ramp.



Force	x	y
F_N	0	F_N
f	$-0.15F_N$	0
mg	$\frac{3}{5}mg$	$-\frac{4}{5}mg$
x-equation: $\frac{3}{5}mg - 0.15F_N = ma$		
y-equation: $F_N - \frac{4}{5}mg = 0$		

$$\frac{3}{5}mg - 0.15\left(\frac{4}{5}mg\right) = ma$$

$$a = .48g = 4.8\,\frac{m}{s^2}$$

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Mass on Ramp Worksheet

A 5 kg mass is placed on a ramp making an angle of 37° w/t the horizontal and released. It slides down the ramp.

1. **What is the acceleration of the mass if the ramp is frictionless?**

2. **What is the acceleration of the mass if the coefficient of friction is 0.2 and the mass is initially sliding down the ramp?**

3. **What is the acceleration of the mass if the coefficient of friction is 0.2 and the mass is initially sliding up the ramp?**

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Mass
on
Ramp
Worksheet

Student Practice
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Bonnie & Clyde Inclined Plane Worksheet

Bonnie and Clyde, the top Physics students in school, designed a Science Fair experiment to determine the coefficients of friction, μ_s and μ_k, between a block and a ramp. The ramp is 2.0 m long, and the mass of the block is 450 g.

1. To determine μ_s, Bonnie placed the block on the ramp. She increased the angle slowly until the block just before it began to move. This angle was measured to be 37°. Determine μ_s from this information.

2. To determine μ_k, Clyde placed the block on the ramp. He increased the angle slowly until the block began to move. But the mass accelerated down the ramp at this angle. He then lowered the angle so that block moved with a constant speed. The angle at which the block moved with constant speed was 25°. Determine μ_k from this information.

3. Their brilliant Physics teacher than challenged Bonnie and Clyde to find the acceleration of block down the ramp that Bonnie and Clyde observed when the block first began to move in the second problem. (What angle was that?)

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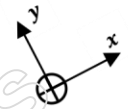
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Bonnie
&
Clyde
Worksheet

Student Practice
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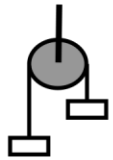
Pulleys

Continuing the discussion around coordinate systems for a bit longer, we have just seen how to handle a ramp situation by rotating the coordinate system as shown at right. But at some point what we usually consider to be the x and y direction becomes unrecognizable. Just rotate the above coordinate system much more and we get see that x is the more vertical variable and y the more horizontal variable. The point is then that what we label the axes is irrelevant.





Let's look at the pulley system at right. If the mass on the right is heavier than the mass on the left, than the right mass accelerates downward and the left mass accelerates upward. But this is then two separate coordinate systems to keep track of. Better would be to consider the direction of the acceleration of the rope. The rope will accelerate clockwise around the pulley – a *single coordinate system* applied to the system. So for the right mass, positive is down, and for the left mass, positive is up.



We can attach a rope to a mass on a horizontal table, over a pulley, and then attached to a hanging mass. For each mass, positive direction would be along the rope; perpendicular to the rope, the forces necessarily cancel.

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Pulleys

In my classes, I typically just give the Basic Pulley Quiz on the next slide to my students to work on in groups without having given any instruction on pulleys – just based on the work already presented. They realize what the motion will be like – that the two masses will "move together" because of the rope, as was the case for the Train with Friction problem earlier.

Typically, they will choose the same coordinate system for both masses, but not realize that the accelerations are in opposite directions. As a result, most students will get an acceleration of $10\ \frac{m}{s^2}$ (which would be free-fall) and a tension in the rope of $100\ N$ or 0, depending on how they do it.

A number of students may recognize that the accelerations of the two masses have "opposite signs" if positive is up for both masses – which is great. However, thinking ahead, when the mass of the pulley must be taken into account and the resulting torques must also be taken into account, even these bright students will end up with inconsistencies.

By focusing on the acceleration of the rope as the positive direction, the positive angular acceleration of the pulley will automatically be consistent with the positive linear acceleration of the rope.

In any event, the student will be primed for new instruction.

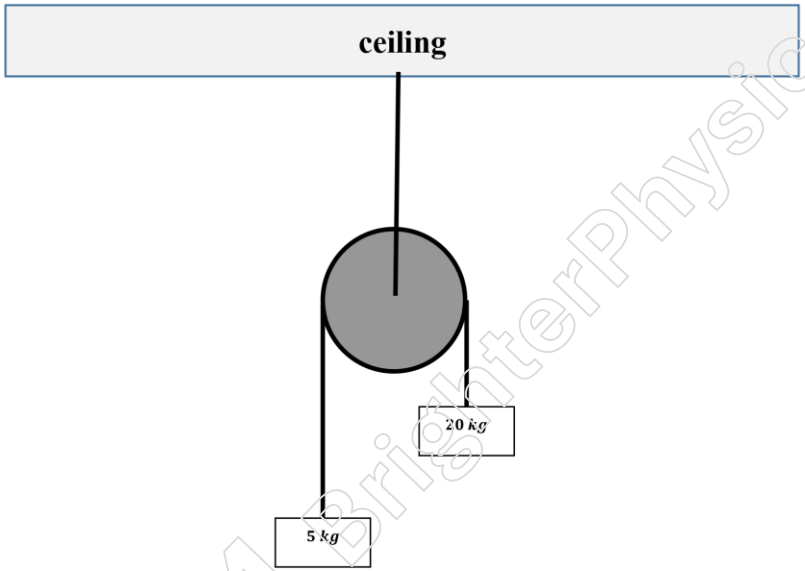
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Basic Pulley Quiz

A massless frictionless pulley is suspended from a ceiling. Two masses, one 5 kg, and the other 20 kg are connected by a massless rope that passes over the pulley.

a) Find the tension in the rope
b) Find the acceleration of the two masses.

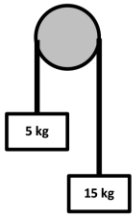


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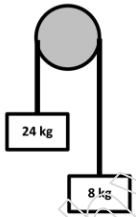
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Pulley Four Square Worksheet

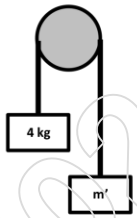
1. Find the tension, T, of the rope.



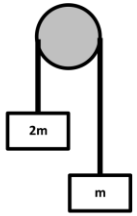
2. Find the magnitude of the acceleration of the masses



3. If the 4 kg mass accelerates downwards at 2.0 m/s^2, find the mass m'.



4. Find the magnitude of the acceleration of the masses



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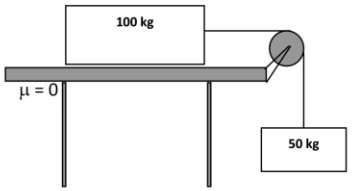
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Pulley
Four
Square
Worksheet

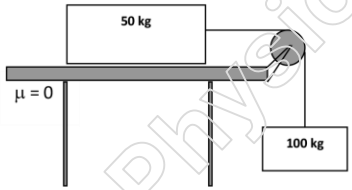
Student Practice
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Table/Pulley Four Square Worksheet

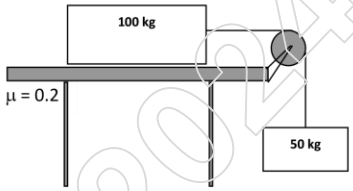
1. Find the acceleration of the two masses.



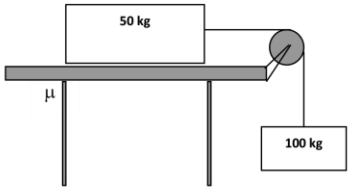
2. Find the acceleration of the two masses.



3. Find the acceleration of the two masses if the two masses are originally moving clockwise.



4. Find μ if the two masses are moving clockwise at a constant speed.



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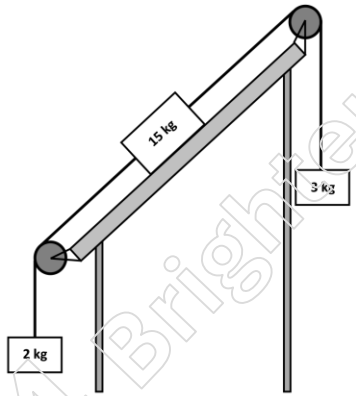
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Table/Pulley Four Square Worksheet

Student Practice Blank

The Most Complicated Pulley Problem Ever Quiz

A **15** $\boldsymbol{kg}$ mass sits on a rough surface having an unknown coefficient of friction $\boldsymbol{\mu}$ and angled at **53°** from the horizontal. This mass is connected by two very light cords that pass over two massless frictionless pulleys, one to the left and one to the right. Hanging freely at the end of these cords is a **2** $\boldsymbol{kg}$ mass to the left and a **3** $\boldsymbol{kg}$ mass on the right. If the masses accelerate at **5.0** $\frac{m}{s^2}$ counterclockwise, find the value of $\boldsymbol{\mu}$.



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The
Most
Complicated
Pulley
Problem
Ever
Quiz

Student Practice
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Centripetal Force

Centripetal Forces

Back in the Introduction to Physics unit, I brought up the issue that students have with the direction of acceleration. There it was noted that students are very familiar with forces, but not so much with acceleration. This observation can be extended a bit.

We interpret the world around us through past experiences. We know from experience that if a force acts on an object, then there will be a change in the velocity, an acceleration, of the object in the direction of that force.

But these formative experiences have all been within an inertial reference frame. An inertial reference frame is one that has zero acceleration, and we have not had much experience with acceleration. We tend to move in straight lines with constant speed. What little acceleration we experience tends to be small (acceleration about the sun or the Earth's rotation) and/or short lived (jumping off a porch).

If we are placed in a non-inertial reference frame, such as a car making a quick turn, our mind interprets what it feels as if we are still in an inertial reference frame. Our mind tells us that we are being thrown from the car – we would have moved radially outward from the center of the circle had it not been for the car door, or seatbelt, or friction - the Centrifugal Force.

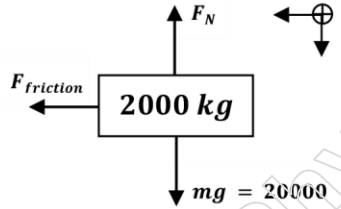
In reality, it is the car door, or seatbelt, or friction, that is pushing us radially toward the center of the circle - the Centripetal Force.

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Centripetal Force

Suppose we have a car of mass m subjected to the constant forces depicted on the Free Body Diagram below.



Following the general procedures given for forces earlier, we would obtain the following table and set of equations.

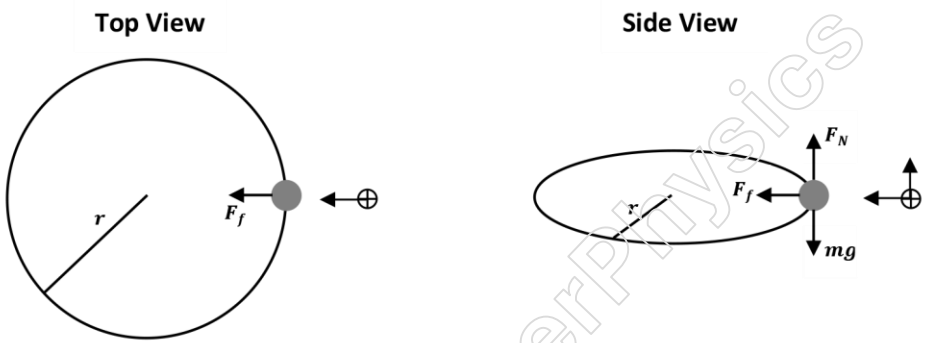
Force	x	y
$F_{friction}$	$F_{friction}$	0
F_N	0	F_N
mg	0	-20000
$F_{net} = ma$	$F_{friction} = ma$	$F_N - 20000 = 0$

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Centripetal Force

Suppose now we have a car of mass $\boldsymbol{m}$ moving in a circle of radius $\boldsymbol{r}$ with *constant speed* $\boldsymbol{v}$. We know that friction is the radial force that keeps the car moving in a circle. There are two FBD that can be drawn:



Knowing that this is a centripetal force problem, we obtain the following table and set of equations following the same force procedures as before.

Force	*radial*	*not radial*
$F_{friction}$	$F_{friction}$	0
F_N	0	F_N
mg	0	-20000
$F_{net} = ma$	$F_{friction} = m\frac{v^2}{r}$	$F_N - 20000 = 0$

The only substantive change is that the acceleration can now be written as $\frac{v^2}{r}$. If the speed is constant then the acceleration is toward the center – the $radial$ direction. To emphasize this, since the coordinate axis is rotating, I label the axis directions $radial$ and $not\ radial$.

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Centripetal Force

The vast majority of centripetal force problems in Physics I assume uniform circular motion where the speed of the object is constant, such as for the previous slide, circular orbits, merry-go-rounds, etc.

The acceleration in these problems, $\frac{v^2}{r}$, is the rate of change in the velocity of the object. But it is the angle that is changing – not the speed. So the acceleration is related to the rate the angle is changing; we need to connect to angular speed. The relationship between the angular and linear speed is $v = r\omega$. Therefore we can write $a = \frac{v^2}{r} = \omega^2 r$.

In the event that the speed is also changing, a pendulum for instance, the instantaneous radial acceleration remains $a_{radial} = \frac{v^2}{r} = \omega^2 r$, but there is also a tangential acceleration $a_{tangential}$ that is equal to the rate at which the magnitude of the velocity changes. In any case, $a_{centripetal} = a_{radial} = \frac{v^2}{r}$.

In the event that the motion is not circular, an elliptical orbit for instance, care must be taken in applying the equation to the orbit. What exactly is "radial" for an elliptical orbit. This is covered in the extension problem Elliptical Orbit Laboratory later in this unit.

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Procedure for Centripetal Problems

Centripetal Summary

Treat centripetal problems like any other force problem, with the following input adaptations:

- Choose positive radially inwards to the center of the circle
- Write replace a_{cent} with $\frac{v^2}{r}$

Force	radial	not radial
$F_{net} = ma$	$F_{cent} = m\frac{v^2}{r}$	$F_{non-radial} = ma_{non-radial}$

The centripetal force is NOT another force. The centripetal force is simply the *sum of the physical forces in the radial direction*. The centripetal force may change as the mass moves – for example, see the Ferris Wheel problem.

The centripetal acceleration IS something. It IS $\frac{v^2}{r}$. If we multiply an acceleration by a mass we get a force. We really should not talk about <u>the</u> centripetal force because it implies that there is some force other than the physical forces acting on the mass.

The non-radial direction may be tangential to the circular path of the mass, as in the Ferris Wheel problem, or it could be perpendicular to the circular path as in the car on a curve problem.

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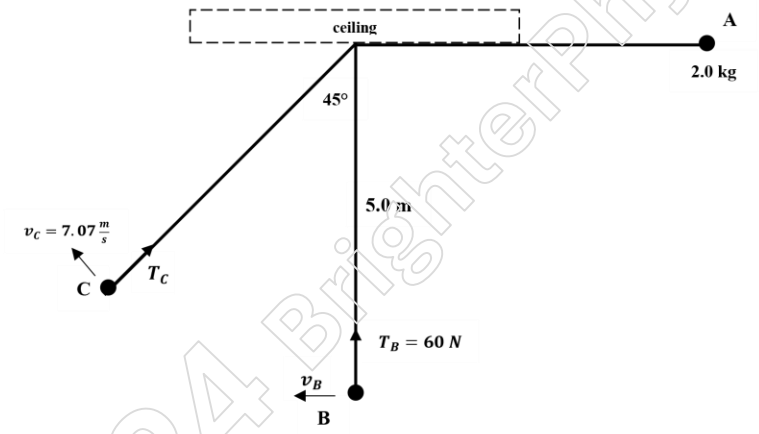
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Pendulum/Centripetal Problem

A $\mathbf{2.0\ kg}$ mass is hung from the end of a $\mathbf{5.0\ m}$ long massless rope. The other end of the rope is attached to the ceiling. We have, therefore, a simple pendulum. The mass is pulled to the side so that there is no slack in the rope and released at Point **A**, so the initial speed is $\boldsymbol{v_A = 0}$.

a. When the mass reaches point **B**, the bottom of the swing, the tension is measured to be $\boldsymbol{T_B = 60\ N}$. Find the speed of the mass at point **B**, $\boldsymbol{v_B}$.

b. At point **C**, where the angle from the vertical is $\mathbf{45°}$, the velocity is $v_C = 7.07\ \frac{m}{s}$. Find the tension $\boldsymbol{T_C}$.



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Centripetal Force Worksheet

1. A runner moving at a speed of 8.8 m/s rounds a bend with a radius of 25 m.
   a) What is the centripetal acceleration of the runner?
   b) What physical force is the centripetal force acting on the runner?
   c) What is the value of the centripetal force if the runner has a mass of 65 kg?

2. A car racing on a flat track travels at 22 m/s around a curve with a 56 m radius.
   a) Find the centripetal acceleration of the car.
   b) What minimum coefficient of friction, μ, between the tires and the road is necessary for the car to round the curve without slipping?

3. An airplane traveling at 201 m/s makes a turn. If the resulting centripetal acceleration is 5 m/s^2, what is the radius of the circular path that the pilot flew?

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Centripetal Force Worksheet page 1

Student Practice Blank

4. A 45 kg high schooler and merry-go-round worker stands on the platform of the ride, 6.3 m from the center. If her speed as she goes around the circle is 4.1 m/s, what is the force of friction necessary to keep her from falling off the platform?

5. A 615 kg racing car completes one lap in 14.3 s around a circular track with a radius of 50.0 m. The car moves at constant speed.
   a) What is the acceleration of the car?
   b) What force must the track exert on the tires to produce this acceleration?

6. If a ring were constructed as part of a space station, what must be the period of rotation of a $50.0\ m$ radius ring rotate to simulate Earth's gravity?

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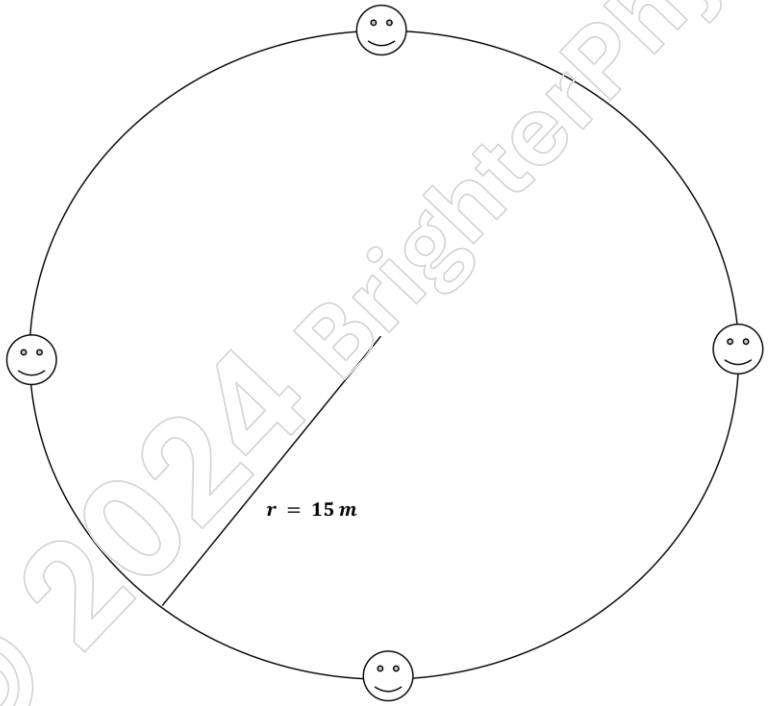
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Centripetal Force Worksheet page 2

Student Practice Blank

Ferris Wheel Worksheet

During Physics Day at a large amusement park, the Ferris Wheel has bathroom scales affixed to the seats. Find the scale reading for a 50 kg rider at each of the four positions indicated. Assume that the Ferris Wheel rotates at constant speed with a period of $T = 31.4\ s$.



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Ferris Wheel Worksheet

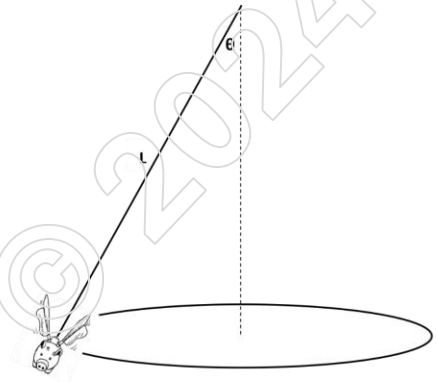
Student Practice Blank

Rotor – Flying Pig Worksheet

There is a carnival ride called the Rotor. The one shown has a radius of 1.5 m. You stand with your back up against the wall. When the ride begins, the wheel spins faster and faster. When it reaches a rotating period of 0.75 seconds, the floor suddenly drops away and there you stay – pinned against the wall. What must be the minimum coefficient of friction to keep you from sliding down?



There is a common Physics lab involving a flying pig. A plastic pig is attached to the ceiling by string of length $\boldsymbol{L}$ and its flapping wings propels it forward. It eventually flies in a circle with a constant speed $\boldsymbol{v}$, the string making an angle of $\boldsymbol{\theta}$ from the vertical direction. Find the speed $\boldsymbol{v}$ of the pig if $\boldsymbol{L = 1.2\ m}$ and $\boldsymbol{\theta = 37°}$.



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Rotor
-
Flying Pig
Worksheet

Student Practice
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Gravitational Force

Newton's Universal Law of Gravity

Universal is a unifying term, not an astronomical one:

- States that the attractive force between astronomical objects is the SAME force that causes apples to fall from trees
- This was not obvious to Newton's contemporaries

The gravitational force:

- Attractive force between objects because of their mass
- force increases with increasing mass
- force decreases as the distance between the masses increase

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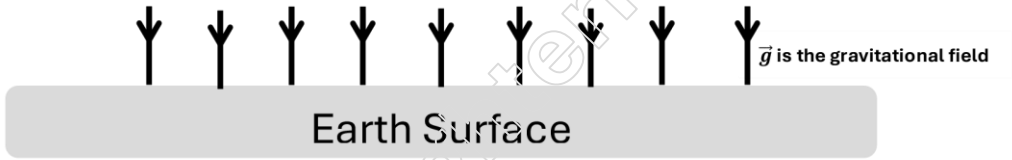
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Gravitational Force

Near Earth's Surface

$$F_g = mg$$

This expression is actually a definition of the Earth's gravitational field near the surface



- An object that has mass is surrounded by a gravitational field
- An object with mass m placed in that field will experience force equal to $m\vec{g}$
- Near to the Earth's surface, that force is $m\vec{g}$ where $\vec{g}$ is $9.8\ \frac{m}{s^2}\ downwards$
- The value of 9.8 is only an average, the actual local value is determined experimentally

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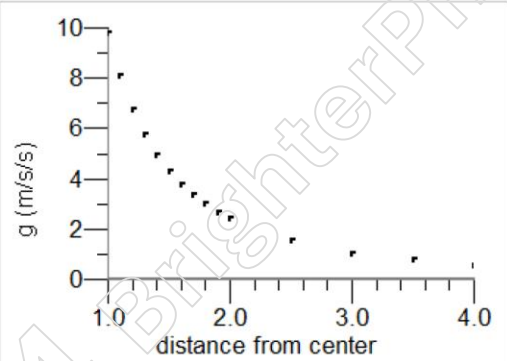
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Gravitational Force

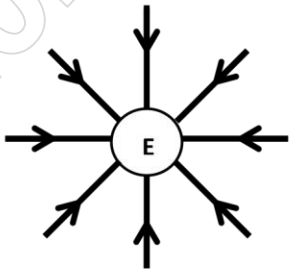
Away from Earth's Surface

What happens to the gravitational field as we move away from the Earth's surface?

Magnitude of the gravitational field as a function of the distance from the Earth's center, in terms of Earth Radii:



Direction of gravitational field away from the Earth:



Radially toward the center

- density of the field lines is greater where the field strength is stronger
- So, magnitude of g is
  - greater near to the Earth
  - smaller farther away

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Gravitational Force

Universal Law of Gravity

$$F_g = G\frac{m_1 m_2}{r^2}$$

- $G = 6.67 \times 10^{-11}\ N \cdot \frac{m^2}{kg^2}$
- m_1 and m_2 are the masses of the two objects
- r is the distance between the centers-of-mass of the two objects
- direction is radially along the line of r
- always an attractive force
- referred to an inverse-square law
  - force is proportional to each mass
  - inversely proportional to the square of the separation

Example:

Gravitational force between two students, 50 kg and 75 kg, whose centers-of-mass are separated by 0.5 m:

$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67\ x\ 10^{-11}\right)\frac{(65)(50)}{(0.5)^2}$$

$$= 8.67\ x\ 10^{-7} N$$

$$= 0.000000867\ N$$

Very small! Static friction with ground is easily large enough to prevent any motion

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Force vs Field

Find the force between Earth and a 65 kg student on Earth's surface:

Universal Law:

$$F = G\frac{m_1 m_2}{r^2}$$
$$= \left(6.67\,x\,10^{-11}\right)\frac{(65)(5.97\times10^{24})}{(6.38\times10^{6})^2}$$
$$= 636\ N$$

Weight:

$$F = mg$$
$$= (65)(9.8)$$
$$= 637\ N$$

The gravitational field is defined by equating these two expressions:

$$G\frac{m_1 m_2}{r^2} = mg$$

Here, m is the mass that is placed into the field g of the other mass

Thus

$$g = G\frac{m}{r^2}$$

This expression is valid for point masses. For extended masses, like the Earth, r is for any point outside of the sphere enclosing the object and centered on the center-of-mass of the object.

In the case of a spherical mass, like the Earth, then for r equal to the radius of the object, g is the gravitational field on the surface. For the surface of the Earth:

$$g = G\frac{m}{r^2} = (6.67\times10^{-11})\frac{5.97\times10^{24}}{(6.38\times10^{6})^2} = 9.78\,\frac{m}{s^2}$$

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Weight of Earth vs Student

A 65 kg student stands on the surface of the Earth:

- **Weight of Student due to Earth a distance R_{Earth} from center:**

$$F = m_{student} g_{Earth}$$

$$= (65\ kg)\left(9.78\ \frac{m}{s^2}\right) = 635.7\ N$$

$$g_{student} = G\frac{m_{student}}{R_E^2}$$

$$= 6.67 \times 10^{-11}\ \frac{65}{(6.38\times10^6)^2}$$

$$= 1.065 \times 10^{-22}\ \frac{m}{s^2}$$

- **Weight of Earth due to Student a distance R_{Earth} from center:**

$$F = m_{Earth} g_{student}$$

$$= \left(5.97 \times 10^{24}\ kg\right)\left(1.065 \times 10^{-22}\ \frac{m}{s^2}\right) = 635.8\ N$$

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Gravity Mini-Lab

Newton's Universal Law of Gravity, $F_{gravity} = G\frac{m_1 m_2}{d^2}$, describes the force between two masses, m_1 and m_2, separated by a distance d between their centers. [Note: mass and distance values can be found on the bottom of this page.]

1. Using the above force law, calculate the gravitational force on a 100 kg mass that is 1, 2, 3, 4, 8, and 16 Earth radii from the center of the Earth. (Note, at 1 Earth radius, the object is on the surface of the Earth!) Enter the values of the distance (in Earth Radii R_E) and the calculated force (in N) in the table below. Show the calculation for 3 Earth Radii distance below the table.

2. Plot the above force (in N) vs. Distance (in Earth radii r_E) on the graph paper provided.

3. Answer the following questions:

   a. Calculate (show work) the gravitational force of the Earth on the Moon?

   b. Calculate the gravitational force of the Moon on the Earth?

   c. Calculate the gravitational force on a 100 kg mass on the surface of the Earth using $F = mg$, where $g = 9.8\ m/s^2$.

   d. How does your answer to question c compare to the value in your table corresponding to the weight on the surface of the earth?

$M_{Earth} = 5.97 \times 10^{24}\ kg$ $M_{Moon} = 7.3 \times 10^{22}\ kg$ $R_{Earth} = 6.38 \times 10^{6}\ m$ $d_{Earth-Moon} = 3.9 \times 10^{8}\ m$

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Gravity Mini-Laboratory page 1

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Gravitational Force (N)	Distance (r_E)
	1
	2
	3
	4
	8
	16

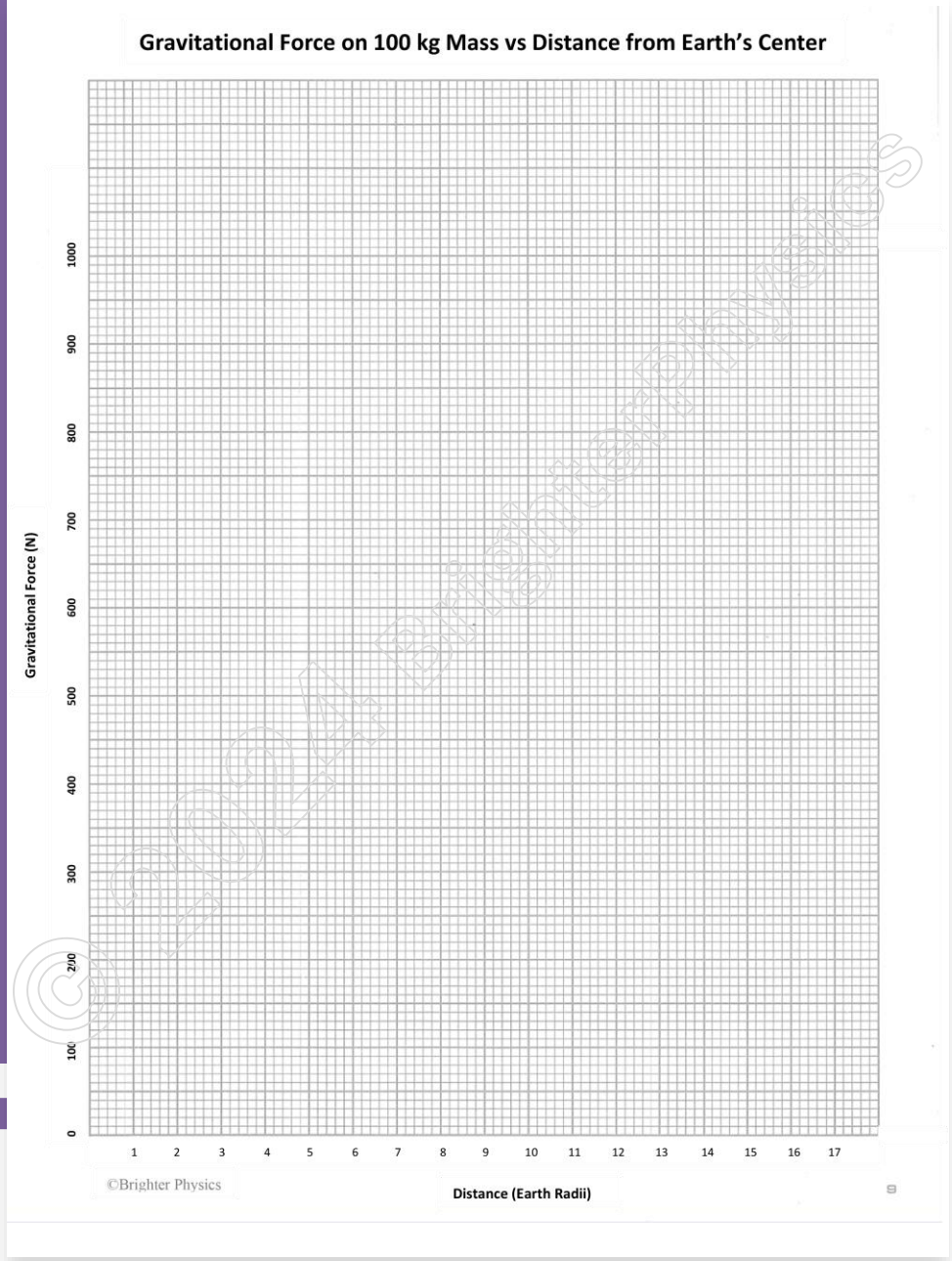
Show the calculation for the force at 3 Earth Radii here:

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Gravity Mini-Laboratory page 2

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Gravity Mini-Laboratory page 3

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Earth-Moon Equilibrium Point

The Earth $(5.97 \times 10^{24} kg)$ and the Moon $(7.34 \times 10^{22} kg)$ are separated by a distance of $(3.9 \times 10^{8} m)$. There is a point between the Earth and Moon where the force on a spacecraft of mass $\boldsymbol{m}$ is zero. How far from the center of the Earth is this point $\boldsymbol{X}$?

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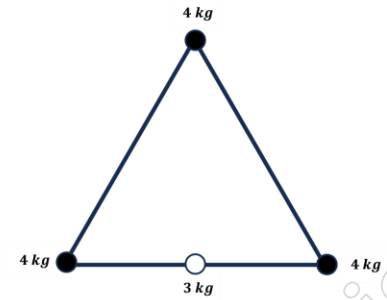
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Earth-Moon Equilibrium Point

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Triangle Gravity Worksheet

A 4.0 kg mass is placed at the corners of an equilateral triangle that has sides of length 2.0 m. Find the gravitational force (magnitude and direction) of a 3.0 kg mass placed at the midpoint of the base.



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Triangle Gravity Worksheet

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Field from v-t Graph Worksheet

A 95 kg astronaut finds herself on an airless moon of Krypton. She knows that the moon has a mass of $7.5 \times 10^{23}\ kg$. To complete her calculation to get off the moon safely, she needs to determine the radius of the moon as well as her weight on the surface of the moon. With her background in Physics she knew that she could determine these if she could produce a velocity-time graph for a falling object. She then measured the velocity of a falling rock and produced the following result. Positive is upwards.



a) What is her weight on the surface of the moon?
b) What is the radius of the moon?

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Field
from
v-t Graph
Worksheet

Student Practice
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Gravity/Circular Motion Worksheet

Assume that the Earth follows a circular path around the sun. And assume 1 year is 365 days. Take the distance from the center of the Earth to the center of the sun to be $1.5 \times 10^{11}\ m$.

Find the mass of the sun from this information.

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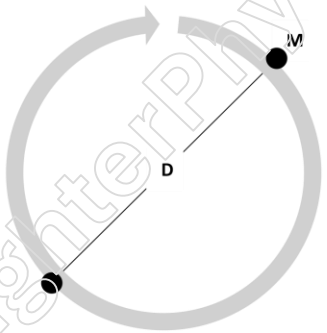
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Gravity
Circular Motion
Worksheet

Student Practice
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Binary Stars Worksheet

Suppose there are two identical stars of mass M separated by a distance D. These two stars orbit about their common center of mass – the center of the circle. Find the speed with which these stars are traveling as they rotate around in terms of G, M, and D.



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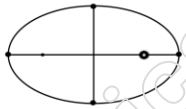
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Binary Stars Worksheet

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Elliptical Orbit Lab

In this lab, we will look at the orbit of a 1.0 kg object around a star of mass 1.49925 x 10^{10} kg. Specifically, you will find the speed of this tiny planet at the four points at the extremes of the orbit – the two farthest and two closest points to the center of the ellipse. These points are shown in the figure to the right. Assume that the planet orbits counterclockwise. (With the above values, this orbit That you will work with is to the actual scale!)



Newton's Universal Law of Gravity, $F_{gravity} = G\frac{m_s m_p}{d^2}$, describes the force between two masses, m_s and m_p, separated by a distance d between their centers.

Since this planet is moving along a curved path, we can also describe it motion using the centripetal force equation, $F_{centripetal} = \frac{m_p v^2}{r}$. The distance r is the radius of curvature of the path, not necessarily the distance between the objects. You should also note that $F_{centripetal}$ is the net force in the radial direction, which is not necessarily $F_{gravity}$.

Directions to create orbit:

- Place two thumbtacks 12 cm apart on the paper provided. This should be at the two positions marked.
- Tie a piece of string into a 32 cm loop. (Hint: attach two nails to a piece of wood to give you the correct length.)
- Loop the string around the thumbtacks and, pulling the string taut with a pencil, draw a complete ellipse. This is the orbit of our hypothetical 1.0 kg planet.
- Remove the thumbtacks and mark the 1.49925 x 10^{10} kg star at one of the foci. (The two focal points are the two holes in the paper you made with the thumbtacks. The given parameters yield $Gm_{star} = 1.0$)

Analysis:

For each of the four points, you are to determine the speed v using the equation $F_{centripetal} = F_{net}^{radial} = \frac{m_p v^2}{r}$. F_{net}^{radial} is $G\frac{m_s m_p}{d^2}$ for the farthest two point, and $G\frac{m_s m_p}{d^2}\cos\theta$ for the closest two points. The angle θ is the angle, measured at the planet from the planet to the center of the ellipse. (The center of the ellipse is the intersection of the two axes in the figure above.)

For each of the four points, find d using a ruler. It is the distance from the star to the planet.
The value for r is the radius of curvature of the ellipse at that point. The two radii of curvatures are .0616 and .1274 m, for the ends of the major and minor axes respectively.

- Enter the values of r and d in the space provided on the orbit handout. Then find the value for the speed v from the resulting equation. Record this value in the space provided.

- Sketch the velocity vector at each of the four locations. (Remember, the velocity vector must be tangent to the curve!) Use the scale of 1 $cm = 1\ \frac{m}{s}$. (That is, if you find speed $v = 2\ \frac{m}{s}$, then draw a vector of length 2 cm.) Assume that the planet orbits counterclockwise.

For the two points at the top and bottom of the orbit, graphically resolve the velocity vector into the component towards or away from the star (v_{radial}), and the component perpendicular to that direction ($v_{tangential}$). The component v_{radial} is a measure of how quickly the planet is approaching or receding from the star, and $v_{tangential}$ is a responsible for the curving of the orbit.

$$m_{star} = 1.49925 \times 10^{10} kg \qquad m_{planet} = 1.0\ kg \qquad G = 6.67 \times 10^{-11}\frac{Nm^2}{kg^2}$$

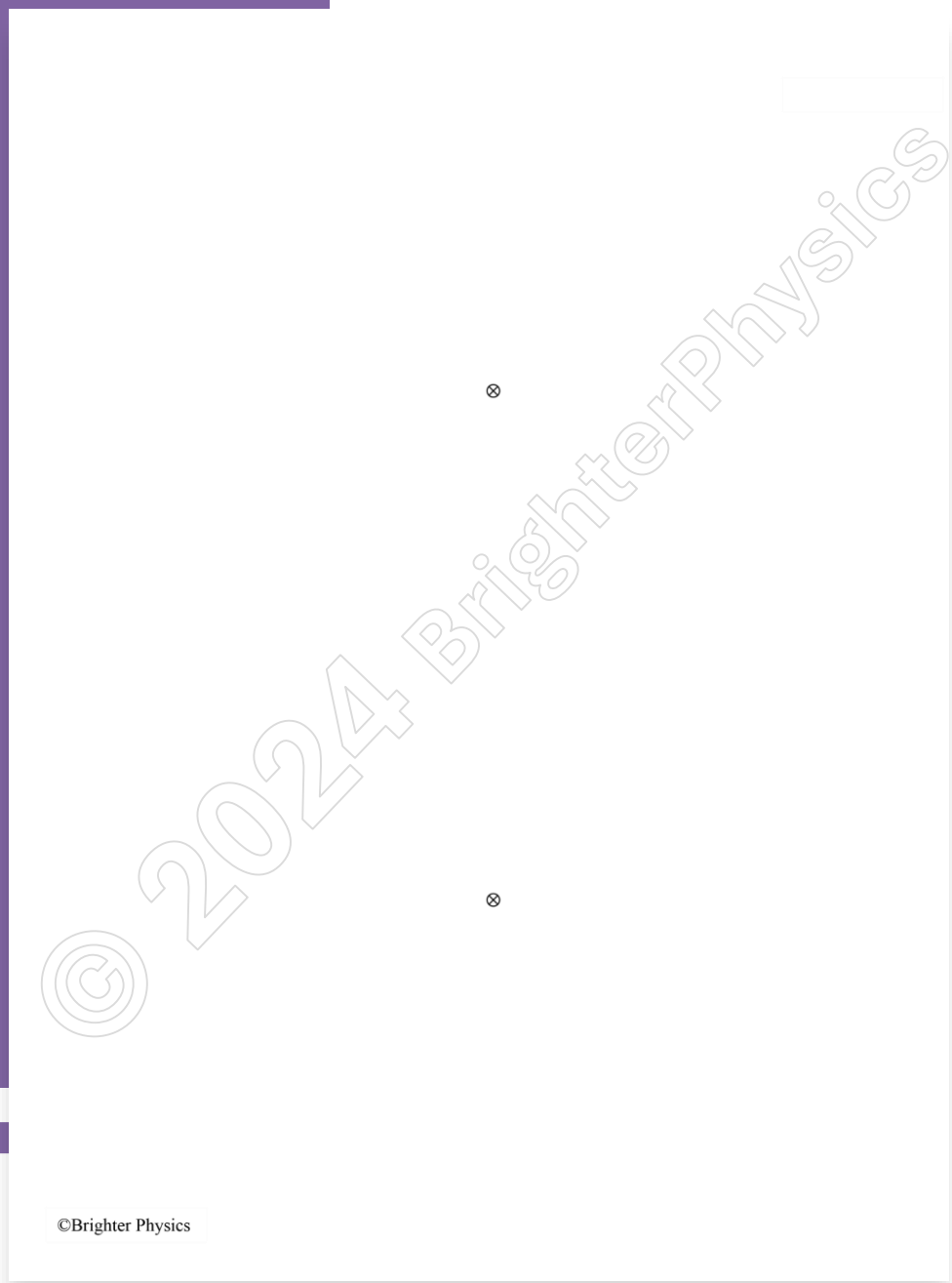
$$F_{gravity} = G\frac{m_s m_p}{d^2} \qquad F_{net}^{radial} = \frac{m_p v^2}{r}$$

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Elliptical Orbit Laboratory page 1

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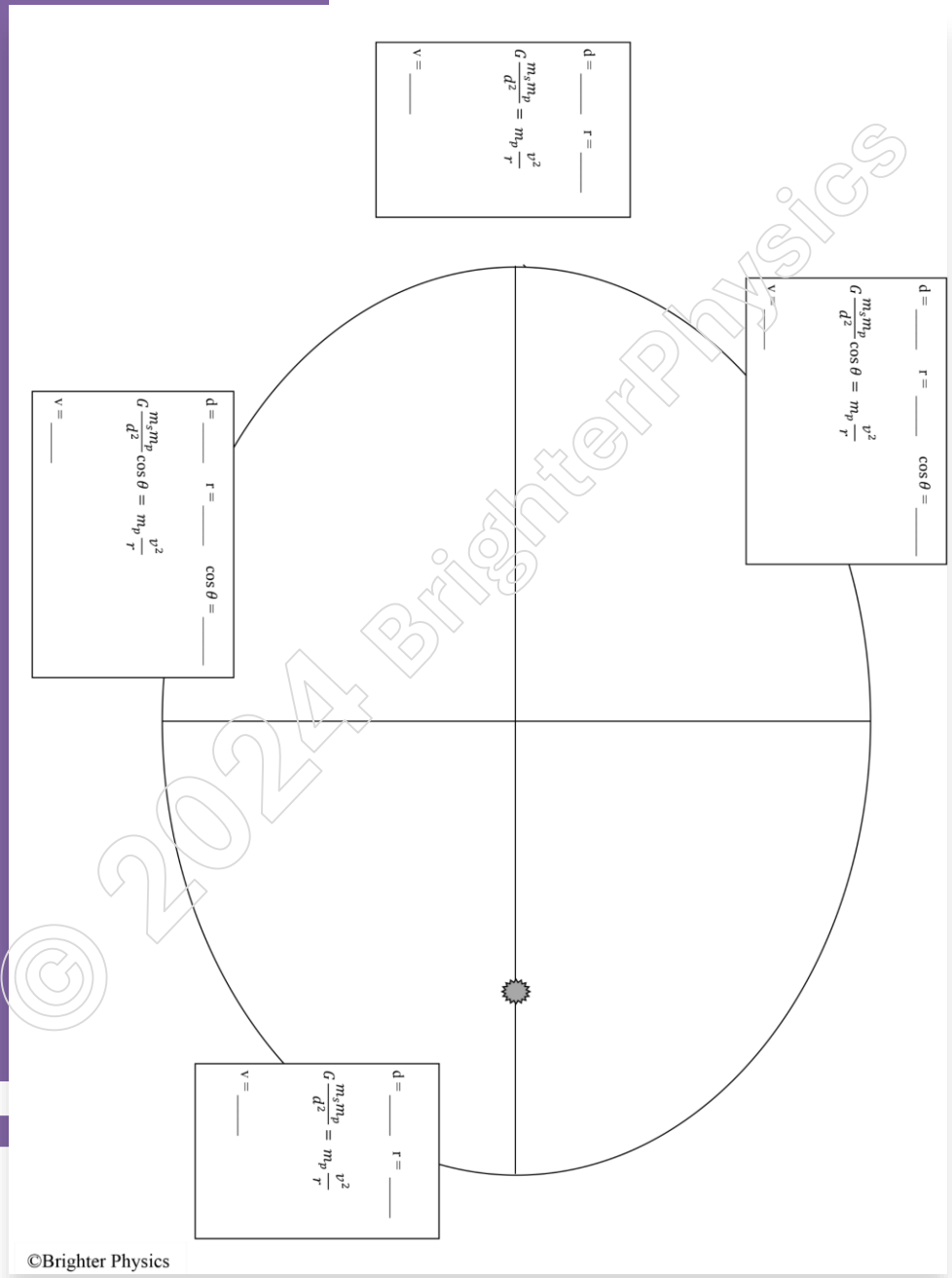


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Elliptical Orbit Laboratory page 2

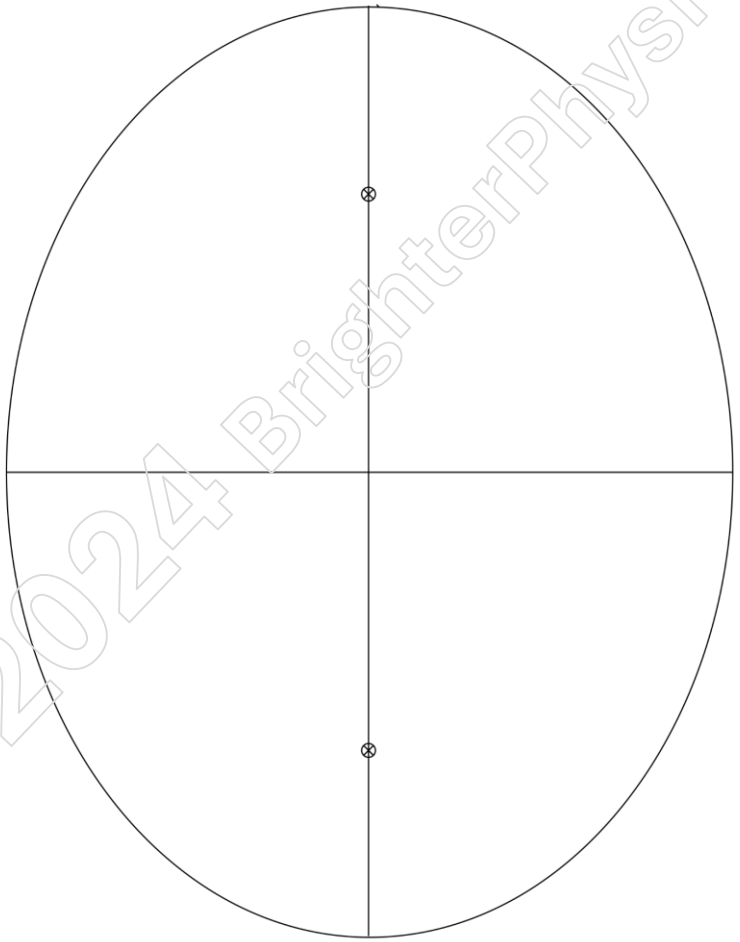
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Elliptical Orbit Laboratory page 3

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Elliptical Orbit Laboratory page 4

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All Your Orbital Needs In One Place

Because orbital motion of a two objects system is such a unique situation compared to the other forces, all the results have been summarized here in one place. This includes momentum and energy considerations also.

For Circular Orbits:

- Since only one force is acting: $\frac{mv^2}{r} = G\frac{mM}{r^2}$, and so we can write $\frac{1}{2}mv^2 = \frac{1}{2}G\frac{mM}{r}$ which relates the kinetic and potential energies of an object of mass m in a circular orbit about a mass M. Therefore $T = -\frac{1}{2}U$. Since $E_{total} = T + U$, we therefore arrive at $E_{total} = \frac{1}{2}U = -\frac{1}{2}G\frac{mM}{r}$.
  - Different orbits for the orbiting mass can then be compared
  - For elliptical orbits, the above relationship for E_{total} is still true if we use the semi-major axis: $E_{total} = -\frac{1}{2}G\frac{mM}{a}$

For Elliptical Orbits:

- $E_{total} = T + U = \frac{1}{2}mv^2 - G\frac{mM}{r}$
  - Different orbital positions within the same orbit can be compared
- For an orbital system of two masses, there is no external force acting
  - linear momentum is conserved
    - So the two masses orbit the center of mass of the system
- The force on the orbiting object is radial, so $\tau_{net} = 0$
  - angular momentum of orbiting object is constant: at apogee $l_a = mv_a r_a$ and at perigee $l_p = mv_p r_p$
    - So $v_a r_a = v_p r_p$

Kepler's Laws:

1st Planets follow elliptical paths

2nd Planetary orbits sweep out equal areas in equal time periods

- Due to Conservation of Angular Momentum: $vr\Delta t = area$

3rd For orbital system: $T^3 \propto r^2$

- See Gravity-Circular Motion Worksheet

Escape Velocity:

The minimum speed needed at launch from the surface of a planet to escape gravitational pull of planet of mass M and radius R:

$$E_{total} = T_\infty + U_\infty = 0 = \frac{1}{2}mv_{escape}^2 - G\frac{mM}{R}$$

$$v_{escape} = \sqrt{2G\frac{M}{R}}$$

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Spring Forces - Hooke's Law

Spring Forces

A spring is any object that will respond to a deformation by "springing back" to it's original configuration, resulting in a **restoring force** on an object. If we push or pull on a spring and deform it, it pushes or pulls back trying to return to its original configuation.

When we imagine a spring in our mind, we typically visualize a metal coil spring, but there are flat leaf springs used as suspension spring in our cars.

Other examples for a spring: a flat ruler; a paper clip; a piece of paper that is softly folder over (not creased) these will spring back when released and are thus springs.

A rubber band can also be considered a spring, but of course a rubber band can only pull, not push.

If you hang a mass from a cord, and then twist the mass clockwise or counterclockwise around the cord and then release the mass, it will rotate back to its original orientation. This would be called a torsional spring.

The undeformed configuration of the spring is called the equilibrium position. The deformation from **equilibrium** is typically written as x, which is often a distance, but could be an angle as for the torsional spring above, among other possibilities.

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Spring Forces - Hooke's Law

Spring Forces

For our purposes, the spring examples in this course will express a linear restoring force. Springs forces that are linear in the deformation are said to obey **Hooke's Law**:

$$F_{spring} = -k(x - x_0)$$

If we think of a coil spring, $(x - x_0)$ is the amount that the spring is compressed or stretched from its equilibrium position, and k is the spring constant, essentially the stiffness of the spring. The negative sign is not to be used literally, which unfortunately many students do. The negative sign means that the force is a restoring force – it is always toward the equilibrium position. Whether that is negative or positive depends on your choice of a positive direction.

Typically, $x_0 = 0$, which simplifies the equation to $F_{spring} = -kx$.

If we write the force as a vector equation, $\vec{F}_{spring} = -k(\vec{x} - \vec{x}_0)$, then the negative sign is necessary – it results in a force that is directed back to the equilibrium position automatically. It is the literal transcription from vector to scalar equation that gives rise to the above difficulty with the negative sign.

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Spring Forces - Hooke's Law

Spring Forces

A spring which follows Hooke's Law, $F_s = -kx$, is called a Hooke's Law Spring, or sometimes an Ideal Spring. Any real spring that satisfies Hooke's Law will only satisfy it within a certain range.

A spring will only "spring back" to its original position only if it has not been permanently deformed in the displacement process. Think of a small paper clip for instance – it is designed to clamp and hold a few sheets of paper. But if you try to clamp too many pages, it has to bend too much and will be permanently deformed. Pulling out the large stack, the paper clip will not go back to being flat. The spring has been deformed past its ***elastic limit.*** If an experiment is performed and the force is plotted against the displacement, where the graph begins to deviate from linearity would mark the elastic limit for that spring. This is investigated in the Rubber Band Hooke's Law Laboratory.

There are springs that do not obey Hooke's Law. Some have been designed to obey a quadratic relationship, for instance. These would not be ideal springs.

The fact that all spring forces, ideal of not, are restoring forces, leads to oscillatory motion – a mass attached to a spring and the spring compressed and released, the mass will oscillate back and forth. If the spring obeys Hooke's Law, the resulting motion is dramatically simplified. The motion will be sinusoidal in time, and the Period of motion will not depend on the displacement.

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Hooke's Law Worksheet

1. A spring stretches 0.12 m when some apples weighing 3.2 N are suspended from it. What is the spring constant of the spring?

2. Each of the coils springs of a car has a spring constant of 25,000 N/m. How much is each spring compressed if it supports ¼ of the car's weight? Mass of car is 4000 kg.

3. A toy rocket launcher contains a spring with a spring constant of 35 N/m. If the toy has a mass of 100 g (look at units!) and the spring is compressed by 30 cm (look at units!) what will be the initial acceleration of the rocket if fired horizontally.

4. What is the mass of a watermelon that stretches a spring scale 0.48 m is the spring constant is 128 N/m?

5. How many centimeters (units!) will a spring scale stretch when a 2.6 kg block is hung vertically from a spring with a spring constant of 89 N/m?

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Hooke's Law Worksheet

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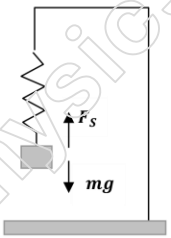
Rubber Band – Hooke's Law Laboratory

Purpose:

To investigate the applicability of Hooke's Law to rubber bands.

Background:

When a mass is attached to a spring and the spring is stretched, it pulls back with a force that is proportional to the stretch. This is called Hooke's Law. Mathematically, it can be expressed as $F_S = -kx$, where k is called the spring constant, F_S is the force exerted by the spring, and x is the stretch *from equilibrium*. The minus sign indicates that the force is always directed back towards the equilibrium point.



The above description of Hooke's Law is only applicable to a limited range in the stretch of the spring. If the initial mass is small, then only the bottom few coils of the spring is engaged. The initial mass should be large enough so that the entire spring (all coils) are involved. It also assumes that the spring is not stretched so much that the spring is permanently deformed.

To the extent that a rubber band acts like a spring, you will investigate the entire stretch range of the rubber band, from too little mass (where only part of the rubber band stretches) through to its ultimate deformation - when the rubber band breaks!

Procedure:

1. Attach a rubber band to any of the long rods around the room.
2. Attach an empty mass holder to the rubber band. Place a $100\ g$ mass on the mass holder. This we call zero mass and zero stretch. Record the position of the bottom of the mass holder.
3. Add $100\ g$ at a time to the mass holder and record the position of the bottom of the mass holder each time, as well as the value of the added mass next to this mark. When adding mass, rotate the added slotted mass so that the bottom of the holder is approximately level.
4. After each mass has been placed onto the holder, the system experiences zero net force, so that the force of the spring is equal in magnitude to the gravitation force on the excess mass added.
5. Record the values of added mass (g) and position (cm) in the table below.
6. After the rubber band breaks, determine the stretch of the rubber band for the added mass. (final position – initial position)
7. Plot appropriately the hanging weight (in N) vs. stretch (in cm).

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Rubber Band
-
Hooke's Law Laboratory
Page1

Student Practice
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Rubber Band – Hooke's Law Laboratory

Added mass (g)	Hanging Weight (N)	Position (cm)	Stretch (cm)
0			0.0

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Rubber Band
-
Hooke's Law
Laboratory
Page2

Student Practice
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Rubber Band – Hooke's Law Laboratory
Teacher Notes

Traditionally, if done as an actual hands-on experiment, the lab usually replicates the lab setup on the Phet simulations platform, using maybe 6 masses and within the linear region of Hooke's Law.

In the lab described here, a rubber band is used in lieu of a metal coil spring. It works best with thinner rubber bands. If the mass is added slowly, as is prescribed here, the kids are amazed at how much the rubber band can hold, how far it can stretch, before it breaks. Thicker rubber bands will usually not have enough room to stretch to the breaking point.

In my classroom, I do not have lab benches, but lab tables – so the height is only about a meter. I have the students put a rubber band on a metal chem lab bench pole. The pole is placed between two lab tables. I have the student raise the pole using 5-6 textbooks to allow for more room.

I use a slotted mass holder. For reproducibility, the students have to sight along the bottom of the mass holder, which means then will end up lying on the floor as the rubber band stretches quite far. (It is also a good place to bring up the idea of parallax in measurements – which will encourage them to get even lower to the floor!

The lab morphs very rapidly into a competition to see which group will hold the most mass. The kids very much enjoy the experience.

Depending on your mass resources, it can be very difficult to get good results primarily because the stack of masses will undoubtedly exceed the length of the mass holder. Often, I will have hooked masses hanging off the sides of the mass holder. Students will swap out 5 100 g masses for a single 500 g mass. To prevent hysteresis effects, they are told to try to keep the position of mass holder steady as they make the substitution.

Ultimately my goal for this lab is to get a reasonable graph from the students so that they can better understand the meaning of Hooke's Law in a real system – why it is valid in some regions by fails in others.

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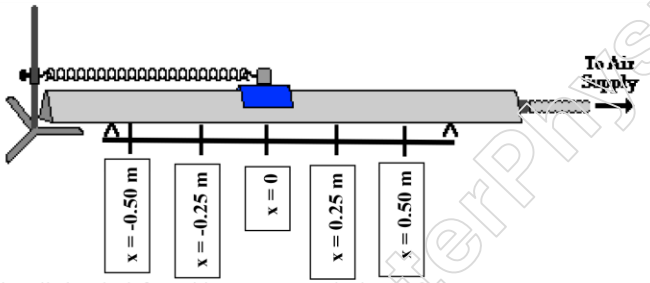
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Rubber Band
-
Hooke's Law Laboratory Teacher Notes

Mass/Spring Acceleration Worksheet

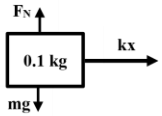
A 100 g mass is placed on a frictionless air track. A spring having a force constant of 1000 N/m is attached to the mass and the edge of the track as shown. The coils of the spring in its equilibrium state are open so that the spring can be compressed as well as stretched. The mass as shown in the picture below is in the equilibrium position (x = 0).

Below the air track is a scale showing the position of the mass. Take positive to be to the right.

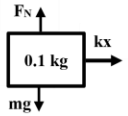


The mass is pulled to the left, position x=-.5 m, and released from rest.

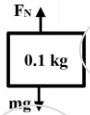
1. What is the acceleration of the mass when x = - 50 m?



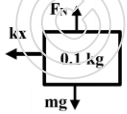
2. What is the acceleration of the mass when x = -.25 m?



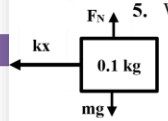
3. What is the acceleration of the mass when x = 0?



4. What is the acceleration of the mass when x = +.25 m?



5. What is the acceleration of the mass when x = +.50 m?



Go to back side.



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Mass/Spring Acceleration Worksheet page 1

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Mass/Spring Acceleration Worksheet

Keeping in mind the acceleration of the mass at the different points in its motion, describe the motion of the mass as a function of time.

In other words, qualitatively describe the velocity of the mass as a function of time after t is released from rest at the point x = -.50 m and then moves to -.25 m, and then to 0.0 m, and so on. What happens to the speed at each of these points?

At x = -.5 m:

At x = -.25 m:

At x = 0:

At x = +.25 m:

At x = +.5 m:

At x = +.25 m:

At x = 0:

At x = -.25 m:

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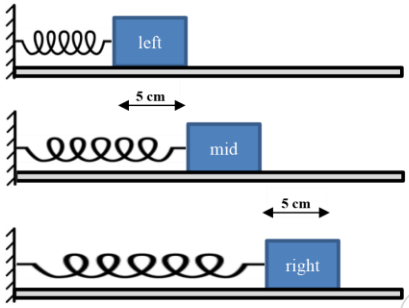
Mass/Spring Acceleration Worksheet page 2

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Multiple Spring Worksheet

A spring of force constant $\boldsymbol{k} = 200\ \frac{N}{m}$ is attached to one end of a horizontal frictionless air track. A $4\ kg$ mass is then attached to the spring, pulled $5\ cm$ and released.

1. Draw the FBD for the midpoint and the two endpoints.



2. What is the net force on the mass at the equilibrium point?

3. What is the magnitude of the net force at each endpoint?

Go to back side.

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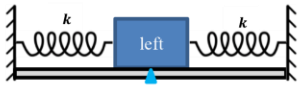
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Multiple Spring Worksheet page 1

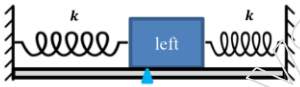
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Multiple Spring Worksheet

4. If a second identical spring is attached to the mass as shown below, what is the effective spring constant, k_{eff}? (Hint: draw the FBD for the mass shown and compare to the FBD for the same mass connected to a single spring having an effective spring constant k_{eff}? How far does each spring stretch or compress compared to the movement of the mass?)



equilibrium position

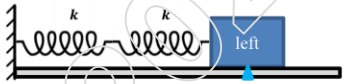


stretched position

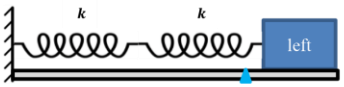


single effective spring

5. If a second identical spring is attached to the mass as shown below, what is the new period of oscillation? (Hint: draw the FBD for the mass shown and compare to the FBD for the same mass connected to a single spring having an effective spring constant $k_{effective}$? How far does each spring stretch or compress compared to the movement of the mass?)



equilibrium position





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