Angular Momentum

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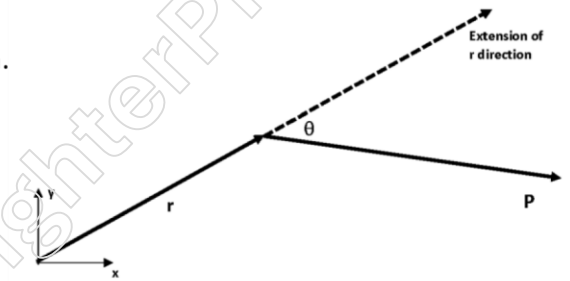
Introduction to Angular Momentum

As with kinematics, angular momentum, can be taught quickly by using the linear analogue.

Linear Motion	Rotational Motion
Momentum $(\vec{P})$	Angular Momentum $(\vec{L})$
"Quantity of Motion"	"Quantity of Angular Motion"
$\vec{p} \equiv m\vec{v}$	$\vec{L} = I\vec{\omega}$
-	$\vec{L} \equiv \vec{r} \times \vec{p}$

Let's look at a general angular momentum problem.

As with all rotational problems, all calculations depend on the choice of coordinate system. Generally, this is at the discretion of the person solving the problem. Choose wisely! Here, the x-y axes are shown at the bottom-left.



The vector $\vec{r}$ is the position of the object with respect to the coordinate system. The vector $\vec{P}$ is the linear momentum of the object at $\vec{r}$. The angle θ is the (smallest) angle between the direction of $\vec{r}$ and $\vec{P}$. Typically, as is the case here, all this is in the plane of the paper/screen.

The direction of the angular momentum will be in the direction perpendicular to the plane formed by the vectors $\vec{r}$ and $\vec{P}$. The direction can be determined from the right-hand-rule, which in this case is into the plane in the negative z direction, or, as expected by the AP, by looking at the rotation of the object w/t the coordinate system – in this case clockwise.

The magnitude of the angular momentum is given by $rP\sin\theta = r_\perp P = rP_\perp$ where $r_\perp$ is the component of $\vec{r}$ in perpendicular to direction of $\vec{P}$, and $P_\perp$ is the component of $\vec{P}$ perpendicular to the direction of $\vec{r}$.

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Introduction to Angular Momentum

Introduction to Angular Momentum

What is Rotational Motion?

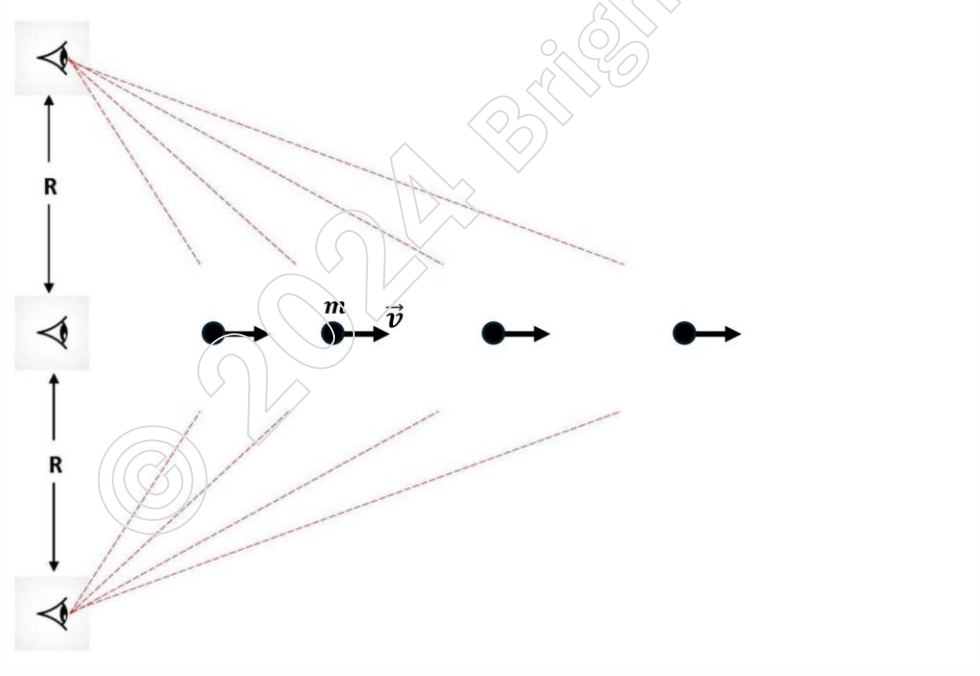
What does it mean for an object to be rotating? This is a different question than asking if an object is spinning. A top spins on its axis, the Earth spins around it axis. Yes, these are also rotations, but a rotation has a more general meaning.

Rotations can be confusing. The Earth rotates around the sun. A mass whirls around out hand as we swing it. We then, incorrectly, assume that an object must follow a curved path if it is to rotate about something.

In the following drawing, a mass $\boldsymbol{m}$ is moving horizontally with a constant velocity $\vec{\boldsymbol{v}}$.

Three different observers, separated by a distance R from each other are shown at left:

- The center observer says that, as time goes on, she does not have to change her line of sight. The mass is not rotating.
- The top observer says that, as time goes on, he has to rotate his line of sight counterclockwise to continue to see the mass. The mass is rotating counterclockwise about him.
- The bottom observer says that, as time goes on, they have to rotate their line of sight clockwise to continue to see the mass. The mass is rotating clockwise about them.

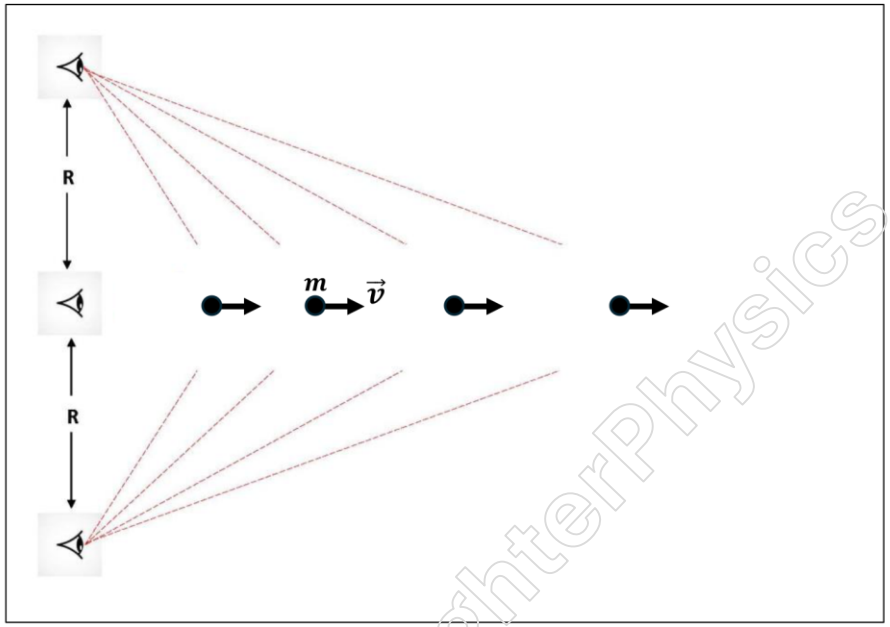


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Introduction to Angular Momentum

What is Rotational Motion?



The mass is moving in a straight line to the right the entire time.

However, each observer, using different coordinate systems (in this case, their own locations) to view the same event, observes distinctly different angular motions. [They each also measure different values for the radial coordinate of $\vec{r}$, by the way. But that has nothing to do with "rotation".]

Looking at the angular momentum measured by the three different observers:

- The center observer says that the angular momentum of the mass is 0 because $\vec{r}$ and $\vec{v}$ are parallel.
- The top observer says that the angular momentum of the mass is counterclockwise with a magnitude of $\boldsymbol{mvR}$.
- The bottom observer says that the angular momentum of the mass is clockwise with a magnitude of $\boldsymbol{mvR}$.

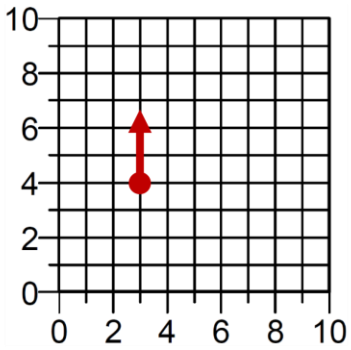
The value of a angular momentum MUST reference the coordinate system

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Angular Momentum Intro Four Square

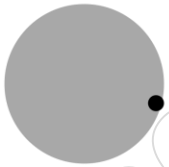
1. A 5 kg mass located at the point (3,4) on the position graph below has a velocity of 12 $\frac{m}{s}$ in the y-direction, shown in red. What is the magnitude of the angular momentum about the origin?



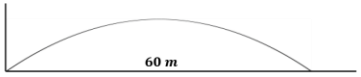
2. A 250 g rock is tied to a string and swung in a horizontal 1.5 m radius circle. The rock makes 5 revolutions in 1 second. What is the magnitude of the angular momentum of the rock about the center of the circle?



3. A 150 kg playground merry-go-round has a radius of 1.5 m. The merry-go-round can be modeled as a uniform circular disk. A 20 kg mass is placed on the rim of the merry-go-round, and the ride spins at a rate of 30 rpm. What is the magnitude of the angular momentum of the system that consists of the ride plus the extra mass?



4. A 0.425 kg soccer ball is kicked at an angle of 37° with a speed of 25 $\frac{m}{s}$. Assume no air friction during the motion of the ball through the air. Assume the origin is the position of the kick.
   a) What is the angular momentum of the ball immediately after kicked?
   b) What is the angular momentum of the ball just before it strikes the ground 60 m from the initial point?



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Angular Momentum Intro Four Square

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Angular Impulse Momentum Theorem

Rotational Impulse-Momentum

Starting from scratch, one could easily and analogously follow the derivation of the impulse-momentum theorem only using angular variables for the purely spinning case; less so for a particle not constrained to a circular path.

Or one could simply cross the position vector into the impulse momentum theorem:

Since $\vec{F}_{net}\Delta t = \Delta\vec{p}$, then

$$\vec{r} \times \vec{F}_{net}\Delta t = \Delta(\vec{r} \times \vec{p})$$

$$\vec{\tau}_{net}\Delta t = \Delta\vec{L}$$

Thus we have the analogous relationships:

Linear	Rotational
$\vec{F}_{net}\Delta t = \Delta\vec{p}$	$\vec{\tau}_{net}\Delta t = \Delta\vec{L}$
$\vec{F}_{net} = \frac{d\vec{p}}{dt}$	$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$

The last equation, of course, is beyond the AP Physics 1 curriculum, although many students will be in, or may have already had, Calculus.

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Angular Impulse Momentum Theorem

$$\tau_{net}\Delta t = \Delta L$$

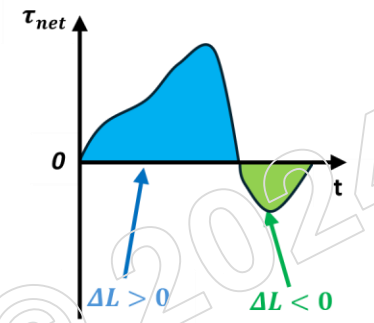
Graphical:

Just as in kinematics, we have this same mathematical structure of the change in two variables in the governing equation, in this case Δt and ΔL. And just like in kinematics, we can look at ways to handle, in this case, graphs of τ or L vs t.

These graphical representations for the Angular Impulse-Momentum Theorem can be compared exactly to the graphical representations for the Impulse-Momentum Theorem.

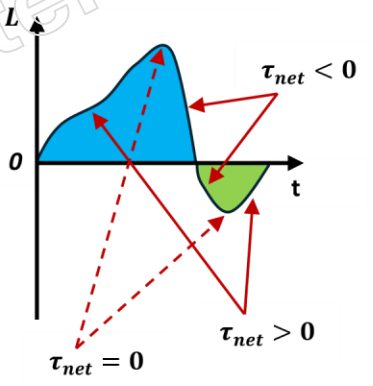
$\tau_{net}\Delta t = \Delta L$

Look at area



$\tau_{net} = \frac{\Delta L}{\Delta t}$

Look at slope



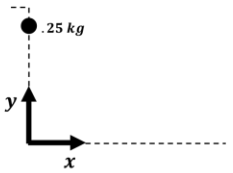
An example of the second graph above can be found in the Torque AP Problem 2021 in the Torque unit.

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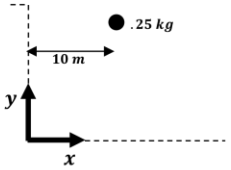
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Angular Impulse Momentum Four Square

1. A 0.25 kg rock is dropped from the edge of a 45 m tall cliff, striking the ground 3 s later. Take as origin the base of the cliff as shown. What is the magnitude of the average torque on the mass as it falls to the ground?



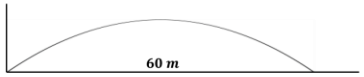
2. A 0.25 kg rock is dropped 10 m from the of a 45 m tall cliff, striking the ground 3 s later. Take as origin the base of the cliff as shown. What is the magnitude of the average torque on the mass as it falls to the ground?



3. A 150 kg playground merry-go-round has a radius of 1.5 m. The merry-go-round can be modeled as a uniform circular disk. A 20 kg mass is placed on the rim of the merry-go-round. If the ride initially spins at a rate of 30 rpm and comes to rest in 15 s, what is the magnitude of the average torque acting on the system causing it to stop rotating?



4. A 0.425 kg soccer ball is kicked at an angle of 37° with a speed of 25 $\frac{m}{s}$. Assume no air friction during the motion of the ball through the air. The ball then strikes the ground 60 m from the initial point 3 s later. Assume the origin is the position of the kick. What is the magnitude of the average torque acting on the ball while it is in the air?



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Angular Impulse Momentum Four Square

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Conservation of Angular Momentum

Conservation of Angular Momentum

For angular momentum to be conserved during some process, then $\boldsymbol{\Delta L = 0}$. So starting from the Angular Impulse Momentum Theorem, $\boldsymbol{\tau_{net}\Delta t = \Delta L}$, angular momentum is conserved if, and only if, $\boldsymbol{\tau_{net}\Delta t = 0}$. For this to be true:

$$\boldsymbol{\tau_{net} = 0} \quad \text{or} \quad \boldsymbol{\Delta t = 0}$$

If $\boldsymbol{\Delta t = 0}$ then there is no event, no process, no problem! In real life macroscopic collisions however, there is often an external force of friction, and therefore an external torque, acting on the masses within the system during the collision. If the collision time, Δt, is small then the angular momentum is approximately true. The issue is that if the external force is a non-conservative force, friction for example, then work is done on or by the system during the collision time. A standard problem might consist of some mass falling onto a spinning disk that may experience friction with the bearings allowing it to spin. During the collision, the falling mass will come to rest with respect to the disk rather quickly and so little energy leaves the system.

There may be internal energy losses, due to friction and deformations **within** the system. This will distinguish between elastic and inelastic collision as we have seen with conservation of linear momentum. These energy considerations are separate from the energy losses due to external forces acting on the system that could invalidate the conservation of angular momentum.

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Spinning Robot Worksheet

A robot stands on freely rotating platform. Initially, with the arms down, the platform rotates with an angular speed of $\omega_i = 0.90 \frac{rev}{s}$. The robot then raises its arms to a horizontal position and as a result the angulsr speed decreases to $\omega_i = 0.60 \frac{rev}{s}$.

a) By what factor does the moment of inertia of the system changed?
b) What is the ratio of the final to initial kinetic energy of the system?
c) Where does the energy difference go or come from?





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Spinning Robot Worksheet

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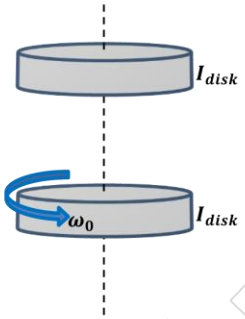
Disk on Disk Worksheet

A cylindrical disk with moment of inertia I_{disk} spins freely with angular velocity ω_0 on a frictionless surface around an axis through its center. A second identical disk initially not rotating is dropped onto the first disk so that their centers line up.

a) Find the angular velocity ω_f of the two disk combined system after this inelastic collision?

b) Why does kinetic energy decreased because of the collision?

c) If the takes time t for the two disk system to come up to speed, what is the average torque exerted on the lower disk in terms of the given variables?



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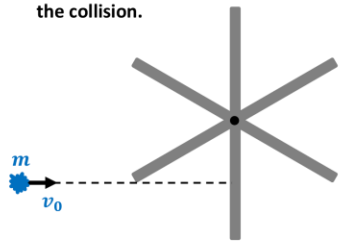
Disk
on
Disk
Worksheet

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Projectile on Disk Worksheet

A paddle wheel like structure consists of 3 uniform rods of mass $\boldsymbol{M}$ and of length $\boldsymbol{2R}$ attached at the center. The wheel can spin freely about a frictionless bearing through its center. It is initially at rest. A sticky wad of mass $\boldsymbol{m}$ and horizontal speed $\boldsymbol{v_0}$ strikes the paddle wheel at the point shown, a distance $\frac{1}{2}\boldsymbol{R}$ from the center. The collision is totally inelastic and consequently the wad and paddle wheel spin as a single unit after the collision. Suppose that $\boldsymbol{M = 10m}$.

a) Find the final angular velocity ω_f of the system about the center of the paddle wheel after the collision.



b) Now, suppose that the collision is not totally inelastic. But rather the wad is somewhat elastic and rebounds with the same speed $\frac{1}{2}v_0$, but in the opposite direction. Find the final angular velocity ω'_f of the system about the center of the paddle wheel after the collision.

c) Why is $\omega'_f > \omega_f$?

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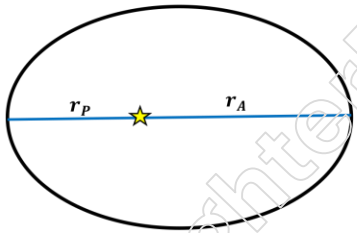
Projectile
on
Disk
Worksheet

Student Practice
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Apogee Perigee Worksheet

The Earth orbits the Sun in an elliptical, not a circular, orbit. At perigee, the Earth is closest to the Sun with a distance of $r_P = 0.9832899\ AU$. At apogee, the Earth is furthest from the Sun with a distance of $r_A = 1.0167103\ AU$.

What is the ratio of the speed of the Earth at perigee to the speed of the Earth at apogee, $\frac{v_P}{v_A}$?



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Apogee
Perigee
Worksheet

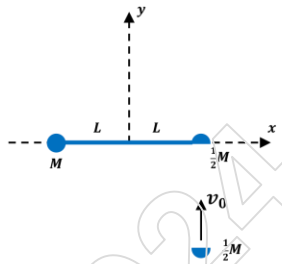
Student Practice
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Space Collision Worksheet

Two small point masses are attached by a **massless** rod of length $2L$. Initially the mass-rod combination is initially motionless in space. Assume a coordinate system where the center of the rod is at the origin and the two masses start on the x-axis at $x = \pm L$. The mass at $x = -L$ has a mass of M and the mass at $x = L$ has a mass of $\frac{1}{2}M$.

A mass $m = \frac{1}{2}M$ having a velocity v_0 in positive $+y$-direction collides with the mass at $x = +L$ at time $t = 0$. Assume the collision takes place rapidly and the collision is totally inelastic. In terms of the given variables

a) Find v_{cm}, the velocity of the center of mass after the collision.
b) Find the angular velocity ω about the center of mass after the collision.
c) Find the tension T in the rod after the collision.



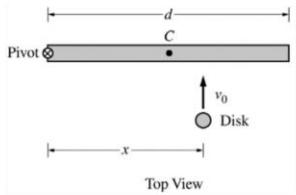
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Space Collision Worksheet

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Angular Momentum AP Problem 2017



Top View

The left end of a rod of length d and rotational inertia I is attached to a frictionless horizontal surface by a frictionless pivot, as shown above. Point C marks the center (midpoint) of the rod. The rod is initially motionless but is free to rotate around the pivot. A student will slide a disk of mass m_{disk} toward the rod with velocity v_0 perpendicular to the rod, and the disk will stick to the rod a distance x from the pivot. The student wants the rod-disk system to end up with as much angular speed as possible.

(a) Suppose the rod is much more massive than the disk. To give the rod as much angular speed as possible, should the student make the disk hit the rod to the left of point C, at point C, or to the right of point C ?

____ To the left of C ____ At C ____ To the right of C

Briefly explain your reasoning without manipulating equations.

(b) On the Internet, a student finds the following equation for the postcollision angular speed ω of the rod in this situation: $\omega = \frac{m_{disk} x v_0}{I}$. Regardless of whether this equation for angular speed is correct, does it agree with your qualitative reasoning in part (a) ? In other words, does this equation for ω have the expected dependence as reasoned in part (a) ?

____ Yes ____ No

Briefly explain your reasoning without deriving an equation for ω.

(c) Another student deriving an equation for the postcollision angular speed ω of the rod makes a mistake and comes up with $\omega = \frac{I x v_0}{m_{disk} d^4}$. Without deriving the correct equation, how can you tell that this equation is not plausible—in other words, that it does not make physical sense? Briefly explain your reasoning.

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Angular Momentum AP Problem 2017 page 1

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Angular Momentum AP Problem 2017

For parts (d) and (e), do NOT assume that the rod is much more massive than the disk.

(d) Immediately before colliding with the rod, the disk's rotational inertia about the pivot is $m_{\text{disk}}\,x^2$ and its angular momentum with respect to the pivot is $m_{\text{disk}}\,v_0\,x$. Derive an equation for the postcollision angular speed ω of the rod. Express your answer in terms of d, m_{disk}, I, x, v_0, and physical constants, as appropriate.

(e) Consider the collision for which your equation in part (d) was derived, except now suppose the disk bounces backward off the rod instead of sticking to the rod. Is the postcollision angular speed of the rod when the disk bounces off it greater than, less than, or equal to the postcollision angular speed of the rod when the disk sticks to it?

____ Greater than ____ Less than ____ Equal to

Briefly explain your reasoning.

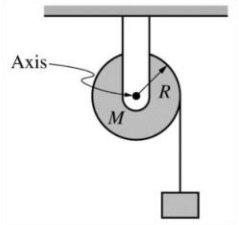
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Angular Momentum AP Problem 2017 page 2

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Angular Momentum AP Problem 2023



A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass M and radius R. The rotational inertia of the pulley is described by the equation $I = \frac{1}{2}MR^2$. The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is F_T.

(a) Determine an expression for the magnitude of the angular acceleration α_D of the disk as the block travels downward. Express your answer in terms of M, R, F_T, and physical constants as appropriate.

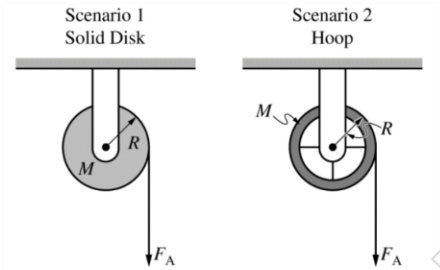
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Angular Momentum AP Problem 2023 page 1

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Angular Momentum AP Problem 2023



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass M and radius R as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force F_A for the same time interval Δt, causing the pulleys to rotate without the string slipping. After time interval Δt, the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval Δt. In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

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