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1-D Kinematics
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The Kinematic Equations

The Kinematic Equations

For an object with constant acceleration:

I. $$v_f = v_i + at$$

II. $$v_f^2 = v_i^2 + 2ad$$

III. $$d = v_i t + \frac{1}{2}at^2$$

IV. $$d = \frac{1}{2}(v_f + v_i)t$$

a = *constant acceleration during the time interval*

t = *time interval for the motion*

d = *displacement of object during time interval t*

v_i = *initial velocity during the time interval*

v_f = *final velocity during the time interval*

Approaching a Kinematic Problem

The standard approach to solve any given Physics problem is to

- Make a sketch of the problem
- Choose the naked equation (the equation as given)
- Dress the equation (insert given values into the naked equation)
- Solve the resultant expression

It is best to put off the algebra until the end of a given problem, as this sequence does – in general students tend to have more issues with algebra than the Physics.

In this slide deck, a working model that can be used to solve any and all 1-D kinematic problems where the acceleration is constant is developed and presented using the above sequence as the kernal of the ultimate procedure.

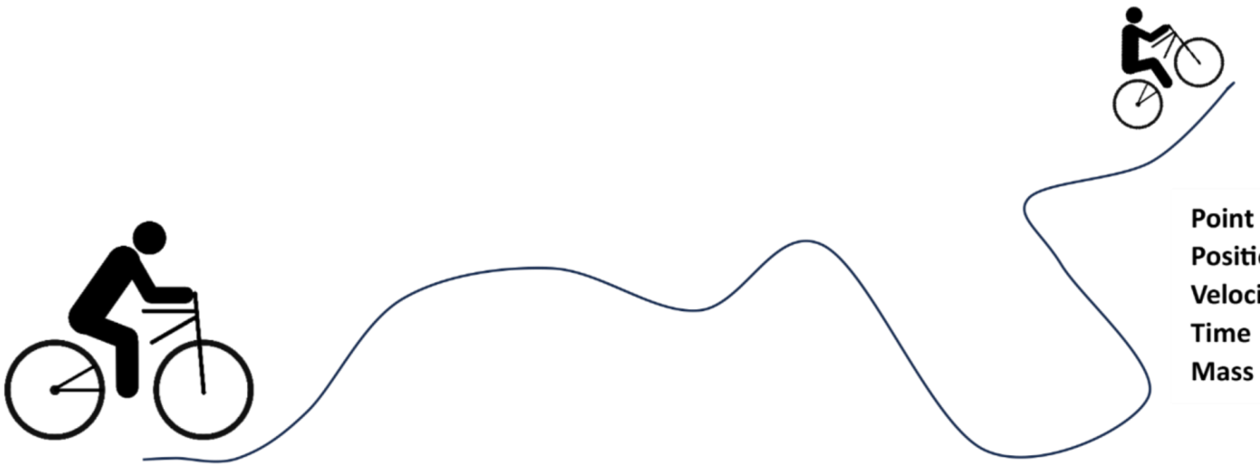
In keeping with the above basic approach to solving Physics problems (sketch, naked equation, dress, solve), we start out with a sketch of the problem. This sketch captures all the relevant information – in particular, all five kinematic variables and their values if known. Each of the five kinematic variables has a specific position in the sketch that are related to their basic meanings.

When motivated properly, the students can themselves generate a very good approximation to what I think an appropriate diagram might look like.

I ask my students to make a sketch of someone riding a bicycle from point A to point B and add to the sketch anything that they think might be important at the starting and ending points of the motion.

An example of something that they might draw is on the next slide.

Sketch of Cyclist from Point A to Point B



Point B
Position
Velocity
Time
Mass

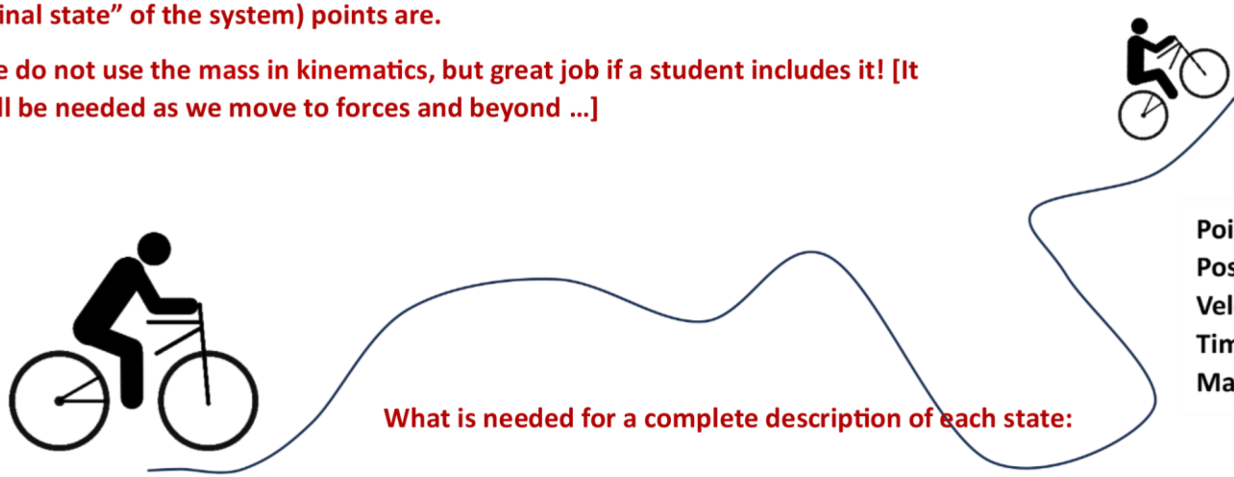
Point A
Position
Velocity
Time
Mass

Sketch of Cyclist from Point A to Point B

This is a good starting point.

It is easy to see where the starting ("initial state" of the system) and ending ("final state" of the system) points are.

We do not use the mass in kinematics, but great job if a student includes it! [It will be needed as we move to forces and beyond …]



Point B
Position
Velocity
Time
Mass

What is needed for a complete description of each state:

Point A
Position
Velocity
Time
Mass

time

position

velocity = rate at which position changes

acceleration = rate at which velocity changes

jerk = rate at which acceleration changes

snap = rate at which jerk changes

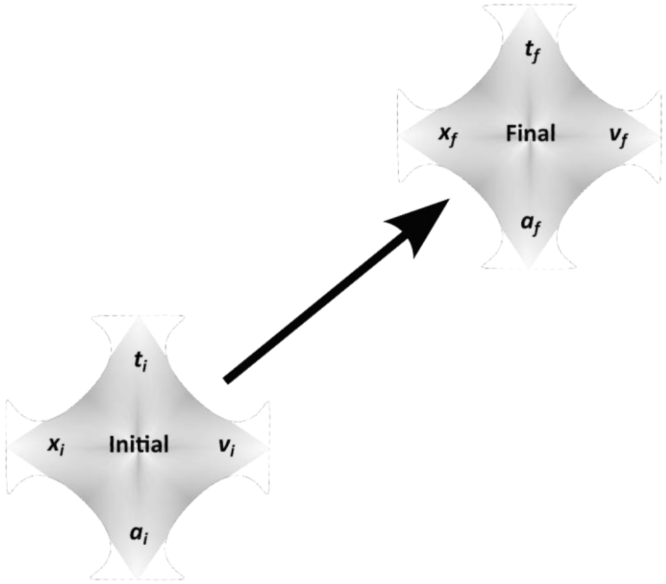
crackle = rate at which snap changes

pop = rate at which crackle changes

…

Approaching a Kinematic Problem

Let's extract the important information for constant acceleration kinematic motion from the drawing of the motion of the cyclist and create the following abstract sketch. Shown in the figure below is the initial and the final states with some sort of transition from the initial state to the final state. [Since the acceleration is constant, the jerk, snapple, crackle, pop, etc. are all zero.]

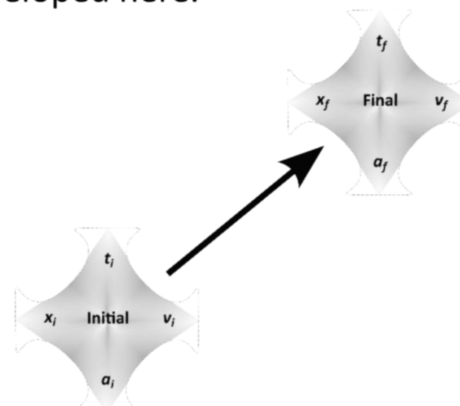


We are not interested in what happens during the motion. We are concerned only with the starting (***i***) and final (***f***) positions, velocities, times, and accelerations.

Approaching a Kinematic Problem

For our purposes, we are interested in motion with constant acceleration, so the acceleration is not associated with the initial and final states separately.

Additionally, we are usually interested in the time interval and the displacement during the motion. The start and final positions and clock times could easily be found, if needed, as an extension to the procedure being developed here.

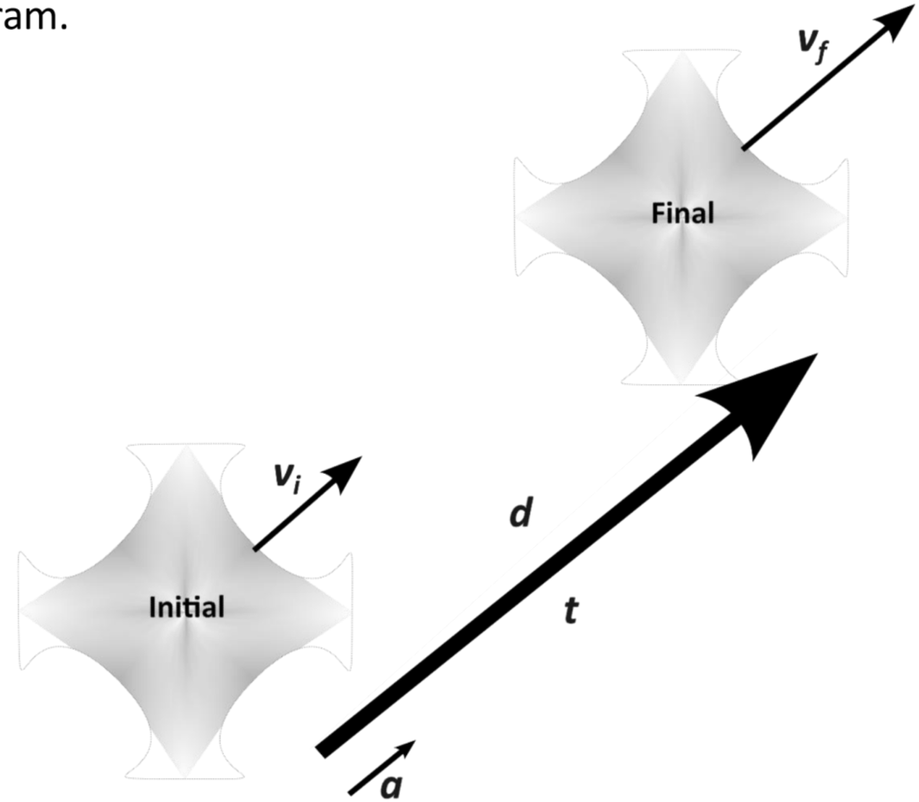


Therefore, the only kinematic variables associated with the initial and final states are the velocities. The time interval, the displacement, and the acceleration are more naturally identified with the arrow that connects the initial and final states.

The arrow itself in this sketch is just symbolically connecting the initial and final states. But in the final diagram on the next slide, it will have morphed into the obvious interpretation of being the displacement vector.

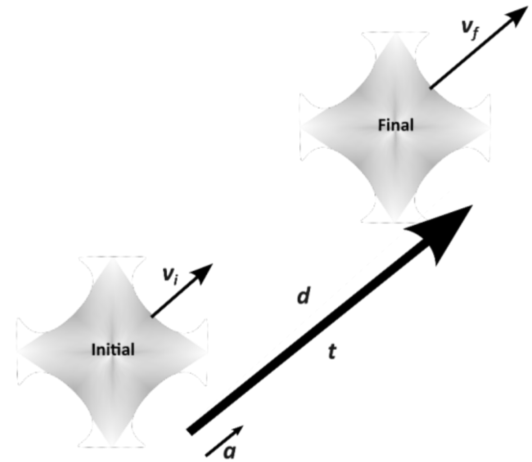
The Kinematic Diagram

Let's call what has been created here the Kinematic Diagram.



It is because of this diagram that I write the kinematic equations in terms of the displacement and time interval rather than the positions and times. The kinematic equations and this diagram are then consistent.

The Kinematic Diagram

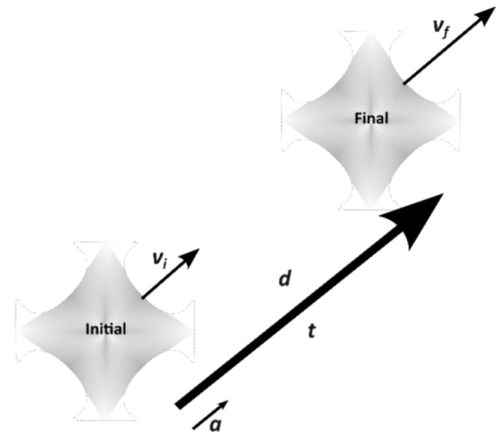


All aspects of a single object undergoing constant accelerated 1-D motion is captured in this single Kinematic Diagram.

The five kinematic variables are all present in this diagram. Four of those variables are vectors, and the direction of each of these vectors are explicitly shown. The initial and final velocities sit on their respective vectors showing the direction of motion, as well as the relative magnitudes. In this case, the object is speeding up.

For problems that involve multiple objects and/or piecewise constant acceleration intervals, multiple kinematic diagrams are constructed and the same procedure to be discussed are applied to each diagram separately.

The Kinematic Diagram



In this diagram, the displacement vector is the arrow that connects the initial and final positions. [This is 1-D motion – the displacement vector is moved so as not to interfere with the velocity vectors that are attached to the initial/final positions.]

The acceleration, in this case, is in the same direction as the displacement as shown by the arrow above the variable.

By definition, the displacement connects the initial to the final position, so the initial and final designations are not needed. However, in all the other procedures that will be presented, the mass of the object will be placed in a box as part of the initial sketch. Having something in the box here increases consistency between the procedures.

Construction of Kinematic Diagram

To codify the steps in constructing a Kinematic Diagram:

- draw initial and final position boxes
- attach initial and final velocity vectors to the boxes and label
- draw and label displacement vector
- place time on displacement vector
- draw and label acceleration vector
- show positive direction

The positive direction can be specified in the problem (implicitly or explicitly) or otherwise chosen by the student. There are two standard choices:

- direction of the velocity (if direction does not change)
- direction of the acceleration

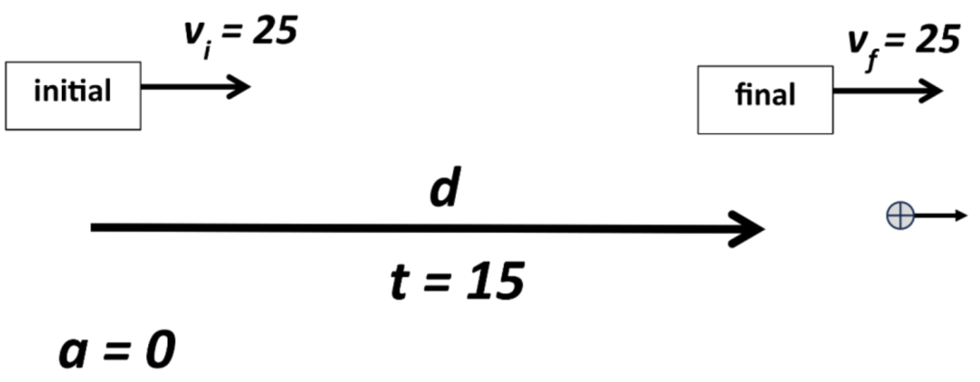
The values of the variables on the diagram really should be the absolute value. This avoids possibly having a negative value along with the arrow pointing in the negative direction which could be confusing. The choice of positive direction really only comes into play when the values are placed into the chosen kinematic equation.

Examples of Kinematic Diagrams

- Example 1

Example 1:

Car moving 25 m/s to right at with constant speed for 15 s:



Suppose we do not realize that the acceleration is zero?

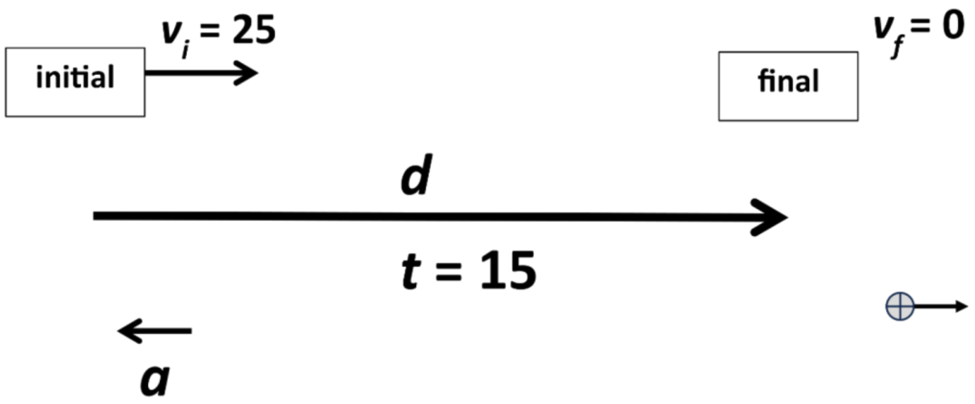
- **The acceleration could be left as an unknown *a* to be found.**
- **We would find a value of 0 using equation I.**

Examples of Kinematic Diagrams

- **Example 2**

Example 2:

Car moving 25 m/s to right uniformly applies brake and stops in 15 s :



We know that the acceleration is toward the left because the car is slowing down.

If asked to find the acceleration:

- **Given this diagram, we would find that acceleration *a* is positive.**
- **If, however, the vector was drawn pointing to the right, we would find that the acceleration is negative. A vector point to the right that is calculated to be negative means it is physically pointing to the left.**

In either case, it is physically to the left.

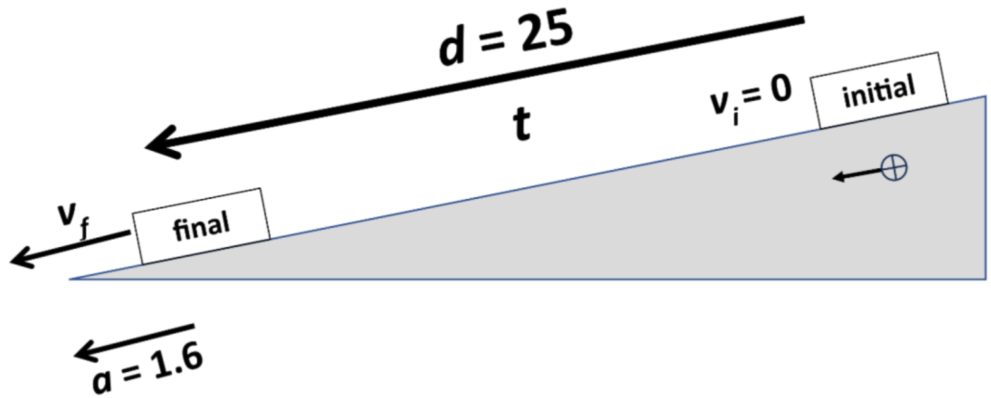
In short, pick a direction if not sure – the solution will let you know which would be the better choice. Neither choice is incorrect. It is simply a matter of interpretation of the mathematical result.

Examples of Kinematic Diagrams

- **Example 3**

Example 3:

Starting from rest, a bike rolls down a straight ramp for 25 m with an acceleration of 1.6 m/s^2:



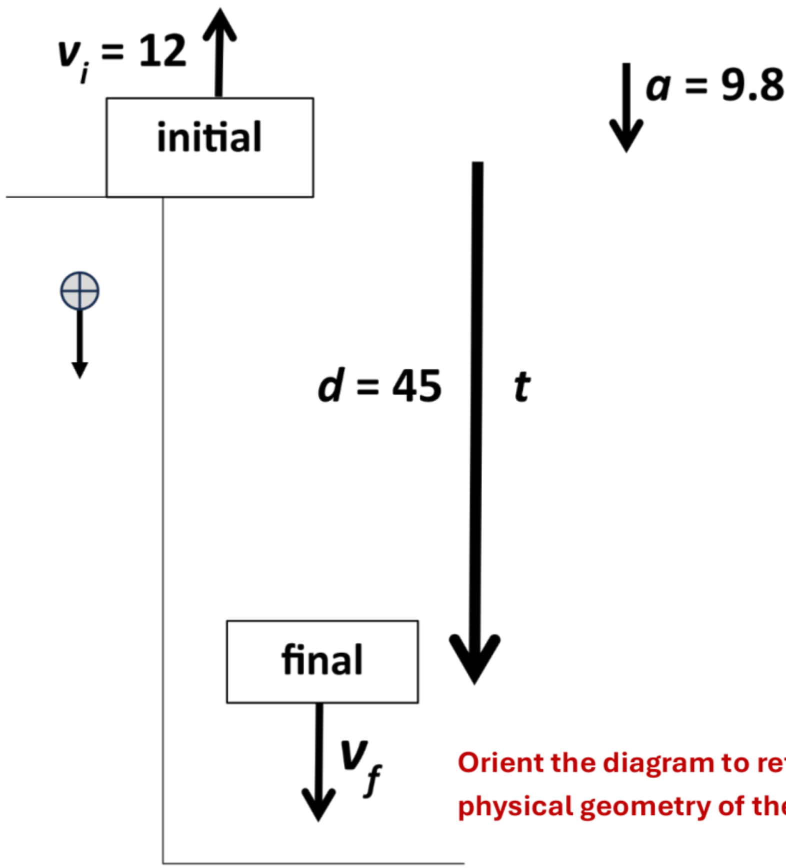
Orient the diagram to reflect the physical geometry of the problem.

Examples of Kinematic Diagrams

- **Example 4**

Example 4:

A student throws a rock upwards with a speed of 12 m/s from the top of a tall cliff. The acceleration is 9.8 m/s^2 downwards. Consider the final position 45 m below the top of the cliff.



Orient the diagram to reflect the physical geometry of the problem.

Kinematic Diagram Four Square - Part 1

Create a Kinematic Diagram for each of the scenarios given.
All 5 variables MUST be present. All 4 vectors MUST be shown. Positive directions MUST be indicated.

1. A car initially moving with a speed velocity of 12 m/s, accelerates for 4.5 s at a rate of 3.7 m/s^2. What is the final velocity of the car?	**2.** A ball is dropped from a tall cliff and falls for 5.1 seconds. The acceleration is 9.8 m/s^2 downward. What is the final speed of the ball?
3. A spacecraft has an initial velocity of +1500 m/s. It fires its engines to slow down to 1100 m/s in the same direction. The magnitude of the acceleration is 19.8 m/s^2. How long did the engines fire?	**4.** You throw a ball downwards with a speed of 2.0 m/s and it falls 2.5 m. The acceleration is 9.8 m/s^2 downward. How fast will it be moving when it strikes the ground?

Kinematic Diagram Four Square Part 1:

Student Practice Blank

Kinematic Diagram Four Square - Part 1

Create a Kinematic Diagram for each of the scenarios given.

All 5 variables MUST be present. All 4 vectors MUST be shown. Positive directions MUST be indicated.

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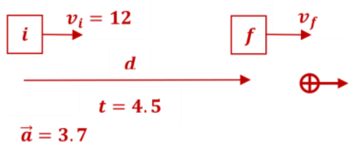
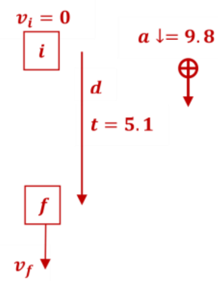


2. A ball is dropped from a tall cliff and falls for 5.1 seconds. The acceleration is 9.8 m/s^2 downward. What is the final speed of the ball?



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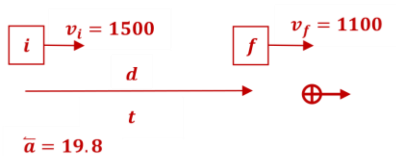
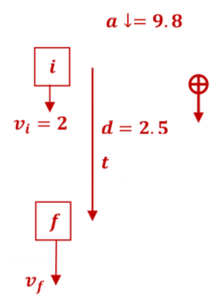


4. You throw a ball downwards with a speed of 2.0 m/s and it falls 2.5 m. The acceleration is 9.8 m/s^2 downward. How fast will it be moving when it strikes the ground?



Kinematic Diagram Four Square Part 1:

Student Practice Solution

Kinematic Diagram Four Square - Part 2

Create a Kinematic Diagram for each of the scenarios given.
All 5 variables MUST be present. All 4 vectors MUST be shown. Positive directions MUST be indicated.

1. A race car has an acceleration of -11 m/s^2 and an initial velocity of +55 m/s. How far will it travel before it stops?	**2.** You drop a ball from the top of the Washington Monument, 169 m above the ground. What is the speed of the ball when it strikes the ground?
3. A car has an initial velocity of +12 m/s and coasts up a hill with an acceleration of -1.6 m/s^2 for 6.0 s. What is the displacement of the car?	**4.** You drop a ball from the top of the Washington Monument, 169 m above the ground. The acceleration of the ball is 9.8 m/s^2. How long does it take for the ball to fall half-way down the monument?

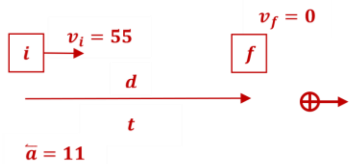
Kinematic Diagram Four Square Part 2:

Student Practice Blank

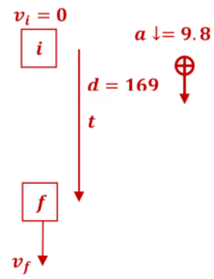
Kinematic Diagram Four Square - Part 2

Create a Kinematic Diagram for each of the scenarios given.
All 5 variables MUST be present. All 4 vectors MUST be shown. Positive directions MUST be indicated.

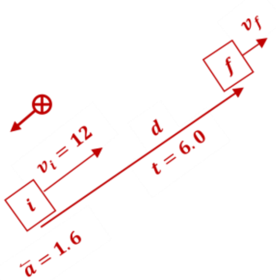
1. A race car has an acceleration of -11 m/s² and an initial velocity of +55 m/s. How far will it travel before it stops?



2. You drop a ball from the top of the Washington Monument, 169 m above the ground. What is the speed of the ball when it strikes the ground?



3. A car has an initial velocity of +12 m/s and coasts up a hill with an acceleration of -1.6 m/s² for 6.0 s. What is the displacement of the car?



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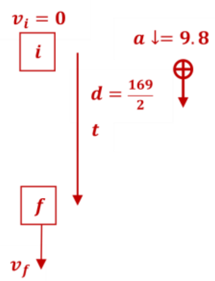


Kinematic Diagram Four Square Part 2:

Student Practice Solution

Kinematic Diagram Four Square - Part 3

Create a Kinematic Diagram for each of the scenarios given.
All 5 variables MUST be present. All 4 vectors MUST be shown. Positive directions MUST be indicated.

1. A car has an initial velocity of 12 m/s and coasts up a hill with an acceleration of -1.6 m/s^2 for 9.0 s. What is its displacement?	**2.** A car has a constant velocity 15 m/s and travels for 6.5 s? How far did it travel?
3. How far does a car that has an initial velocity of +10 m/s and accelerates to a final velocity of +15 m/s travel in 6.5 s?	**4.** How far does a plane travel while flying for 15 s as its velocity changes from 145 m/s to 75 m/s at a uniform rate of acceleration?

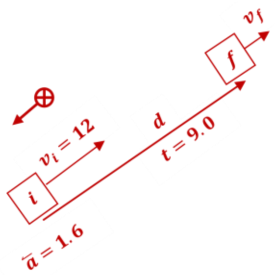
Kinematic Diagram Four Square Part 3:

Student Practice Blank

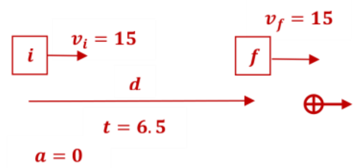
Kinematic Diagram Four Square - Part 3

Create a Kinematic Diagram for each of the scenarios given.
All 5 variables MUST be present. All 4 vectors MUST be shown. Positive directions MUST be indicated.

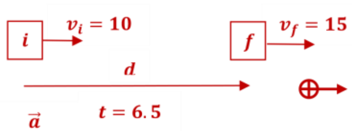
1. A car has an initial velocity of 12 m/s and coasts up a hill with an acceleration of -1.6 m/s^2 for 9.0 s. What is its displacement?



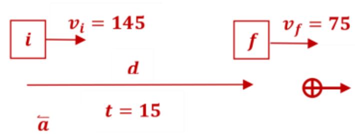
2. A car has a constant velocity 15 m/s and travels for 6.5 s? How far did it travel?



3. How far does a car that has an initial velocity of +10 m/s and accelerates to a final velocity of +15 m/s travel in 6.5 s?



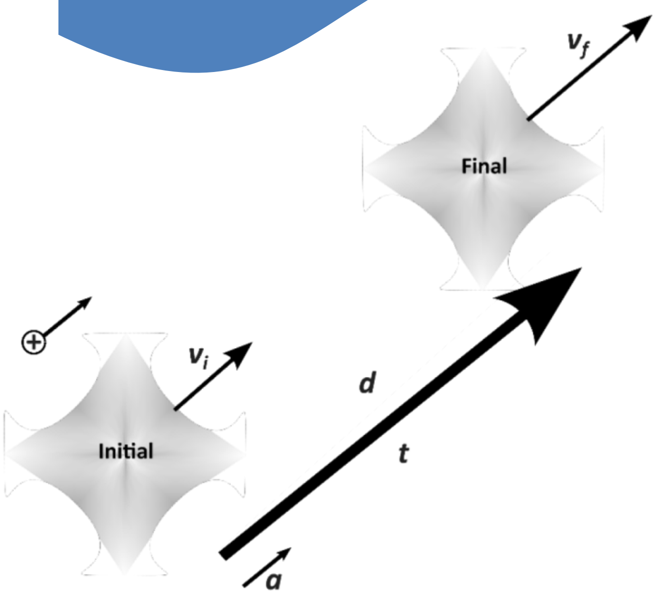
4. How far does a plane travel while flying for 15 s as its velocity changes from 145 m/s to 75 m/s at a uniform rate of acceleration?



Kinematic Diagram Four Square Part 3:

Student Practice Solution

General Procedure for any 1-D Kinematic Problem



General Procedure for any 1-D Kinematic Problem

1. **Draw Kinematic Diagram**
   - based on diagram at left
2. **Choose Naked Equation**
   - which of the four kinematic equations to use
3. **Dress Equation**
   - with variables in the diagram
   - values are positive or negative based on the choice of positive direction
4. **Solve**
   - if "best" equation was chosen, finished
   - if not, a second equation must be chosen - simultaneous equations

1-D Kinematic Example Solutions

- **Intro**

For 1-D kinematic problems, students do not have to figure out a solution to the problem. Students simply need to follow the procedure – the procedure will solve the problem for them.

For each of the four example kinematic diagrams presented above, a question is added and a solution provided using the given procedure and presented in the following few slides.

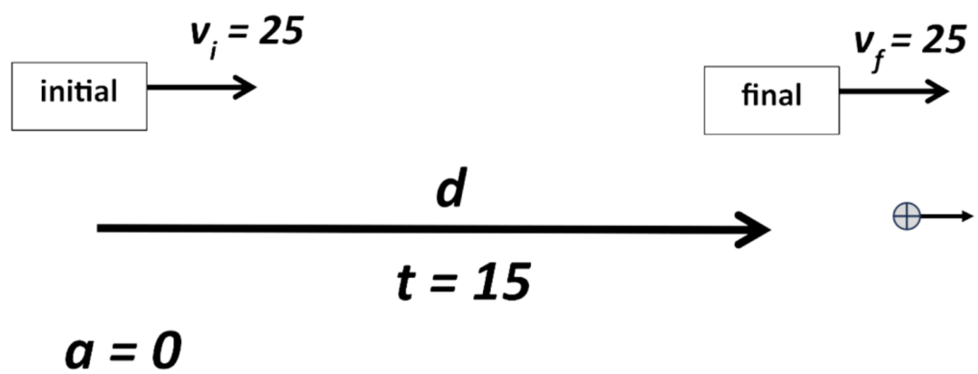
Understanding of the Physics comes with practice, especially after kinematic problems with multiple objects and/or piecewise constant accelerations. Recognition of the validity of the procedure only under constant acceleration enables students to ultimately attack more complex problems by recognizing how to break them down into a connected set of simple problems that they already know how to solve.

1-D Kinematic Example Solutions

- **Example 1**

Example 1:

Car is moving 25 m/s to right at with constant speed for 15 s. What is its displacement?



$$d = \frac{1}{2}(v_f + v_i)t$$

$$d = \frac{1}{2}(25 + 25)15$$

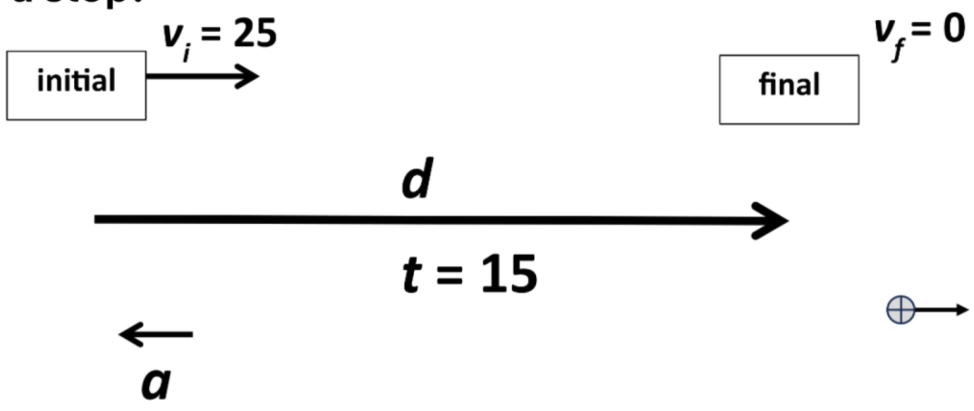
$$d = 375 \text{ m}$$

1-D Kinematic Example Solutions

- **Example 2**

Example 2:

Car moving 25 m/s to right uniformly applies brake and stops in 15 s. What is the acceleration of the car as it comes to a stop?



$$v_f = v_i + at$$

$$0 = 25 + (-a)15$$

$$a = 1.7\ \frac{\text{m}}{\text{s}^2}$$

There is sometimes confusion about the signs. A full explanation of the issue is presented on the next slide.

1-D Kinematic Example Solutions

- **Example 2a**

There is often confusion concerning the sign of an unknown variable when it is pointing in the negative direction. This confusion arises from the fact that the kinematic equations are actually vector equations but are rarely explicitly written that way.

The kinematic equations should be more properly written as:

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

$$v_f^2 = v_i^2 + 2\vec{a} \cdot \vec{d}$$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)t$$

On the previous slide, the naked equation really should read as:

$$\vec{v}_f = \vec{v}_i + \vec{a}t$$

that, as a scalar equation, becomes

$$v_f = v_i - at$$

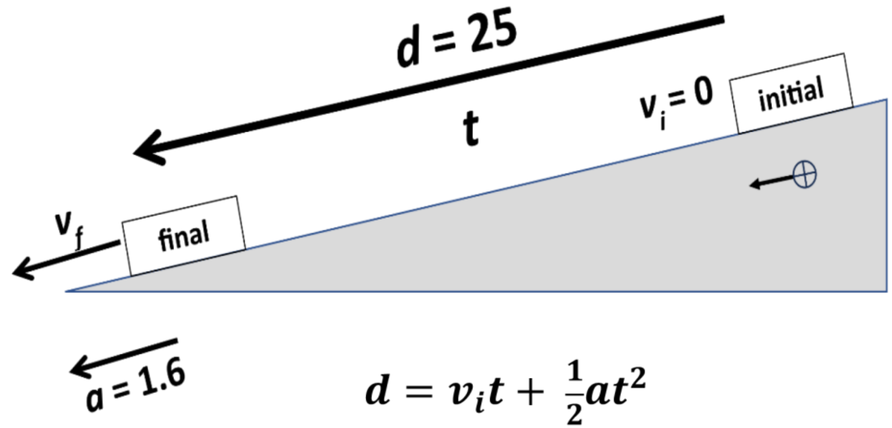
since $\vec{a} = -a\hat{\imath}$, $\vec{v}_i = v_i\hat{\imath}$, and $\vec{v}_f = v_f\hat{\imath}$, where $\hat{\imath}$ is the unit vector in the positive direction and cancels out of the equation.

1-D Kinematic Example Solutions

- **Example 3**

Example 3:

Starting from rest, how long does it take for a bike to roll 25 m down a hill if the acceleration is 1.6 m/s^2?



$$d = v_i t + \frac{1}{2}at^2$$
$$25 = (0)t + \frac{1}{2}(1.6)t^2$$
$$t = 5.6\ s$$

Now, suppose the question was to find the final velocity instead of the time and the above equation was chosen – not the “best equation”.

At this point we can then choose any equation involving v_f, since we know everything else. For example:

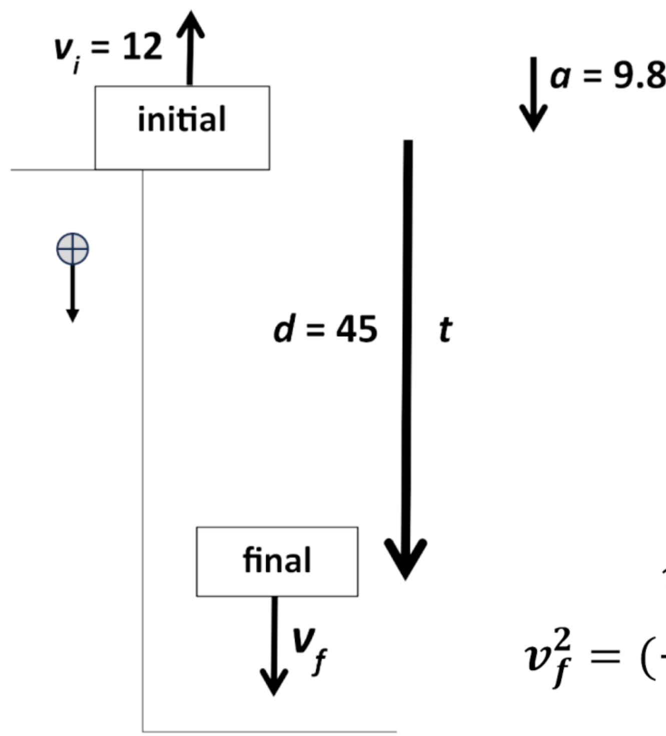
$$v_f = v_i + at$$
$$v_f = 0 + (1.6)(5.6)$$
$$v_f = 9.0\,\frac{m}{s}$$

1-D Kinematic Example Solutions

- **Example 4**

Example 4:

A student throws a rock upwards with a speed of 12 m/s from the top of a tall cliff. The acceleration is 9.8 m/s^2 downwards. What is the speed of the rock when it has fallen 45 m below the edge of the cliff?



$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = (-12)^2 + 2(9.8)(45)$$

$$v_f = 32\frac{m}{s}$$

Kinematic Procedure Four Square - Part 1

Create a Kinematic Diagram for each of the scenarios given.
Choose the appropriate equation and solve for the requested variable.

1. A car initially moving with a speed velocity of 12 m/s, accelerates for 4.5 s at a rate of 3.7 m/s^2. What is the final velocity of the car?	**2.** A ball is dropped from a tall cliff and falls for 5.1 seconds. The acceleration is 9.8 m/s^2 downward. What is the final speed of the ball?
3. A spacecraft has an initial velocity of +1500 m/s. It fires its engines to slow down to 1100 m/s in the same direction. The magnitude of the acceleration is 19.8 m/s^2. How long did the engines fire?	**4.** You throw a ball downwards with a speed of 2.0 m/s and it falls 2.5 m. The acceleration is 9.8 m/s^2 downward. How fast will it be moving when it strikes the ground?

Kinematic Procedure Four Square Part 1:

Student Practice Blank

Kinematic Procedure Four Square - Part 1

Create a Kinematic Diagram for each of the scenarios given.
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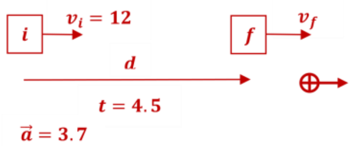
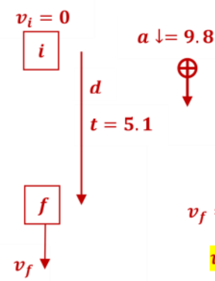


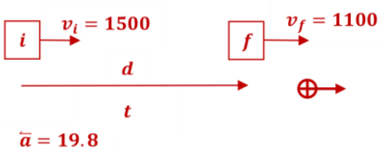
$$v_f = v_i + at$$
$$v_f = 12 + (3.7)(4.5)$$
$$v_f = 28.7 \text{ m/s}$$

2. A ball is dropped from a tall cliff and falls for 5.1 seconds. The acceleration is 9.8 m/s^2 downward. What is the final speed of the ball?



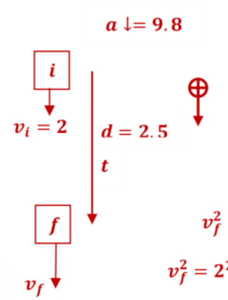
$$v_f = v_i + at$$
$$v_f = 0 + (9.8)(5.1)$$
$$v_f = 50.0 \text{ m/s}$$

3. A spacecraft has an initial velocity of +1500 m/s. It fires its engines to slow down to 1100 m/s in the same direction. The magnitude of the acceleration is 19.8 m/s^2. How long did the engines fire?



$$v_f = v_i + at$$
$$1100 = 1500 - (19.8)t$$
$$t = 20.2 \text{ s}$$

4. You throw a ball downwards with a speed of 2.0 m/s and it falls 2.5 m. The acceleration is 9.8 m/s^2 downward. How fast will it be moving when it strikes the ground?



$$v_f^2 = v_i^2 + 2ad$$
$$v_f^2 = 2^2 + 2(9.8)(2.5)$$
$$v_f = 7.28 \text{ m/s}$$

Kinematic Procedure Four Square Part 1:

Student Practice Solution

Kinematic Procedure Four Square - Part 2

Create a Kinematic Diagram for each of the scenarios given.
Choose the appropriate equation and solve for the requested variable.

1. A race car has an acceleration of -11 m/s^2 and an initial velocity of +55 m/s. How far will it travel before it stops?	**2.** You drop a ball from the top of the Washington Monument, 169 m above the ground. What is the speed of the ball when it strikes the ground?
3. A car has an initial velocity of +12 m/s and coasts up a hill with an acceleration of -1.6 m/s^2 for 6.0 s. What is the displacement of the car?	**4.** You drop a ball from the top of the Washington Monument, 169 m above the ground. The acceleration of the ball is 9.8 m/s^2. How long does it take for the ball to fall half-way down the monument?

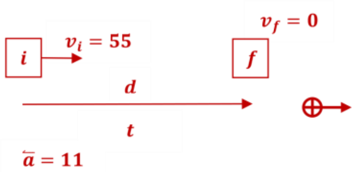
Kinematic Procedure Four Square Part 2:

Student Practice Blank

Kinematic Procedure Four Square - Part 2

Create a Kinematic Diagram for each of the scenarios given.
Choose the appropriate equation and solve for the requested variable.

1. A race car has an acceleration of -11 m/s² and an initial velocity of +55 m/s. How far will it travel before it stops?

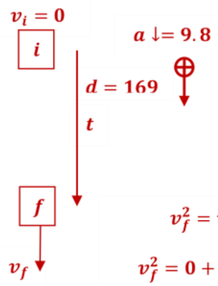


$$v_f^2 = v_i^2 + 2ad$$

$$0 = (55)^2 - 2(11)d$$

$$d = 138\text{ m}$$

2. You drop a ball from the top of the Washington Monument, 169 m above the ground. What is the speed of the ball when it strikes the ground?

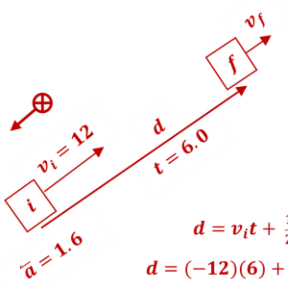


$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 0 + 2(9.8)(169)$$

$$v_f = 57.6\text{ m/s}$$

3. A car has an initial velocity of +12 m/s and coasts up a hill with an acceleration of -1.6 m/s² for 6.0 s. What is the displacement of the car?



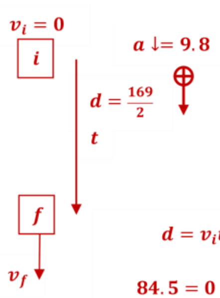
$$d = v_i t + \frac{1}{2}at^2$$

$$d = (-12)(6) + \frac{1}{2}(1.6)6^2$$

$$d = -43.2\text{ m}$$

This is up the ramp.

4. You drop a ball from the top of the Washington Monument, 169 m above the ground. The acceleration of the ball is 9.8 m/s². How long does it take for the ball to fall half-way down the monument?



$$d = v_i t + \frac{1}{2}at^2$$

$$84.5 = 0 + \frac{1}{2}(9.8)t^2$$

$$t = 4.15\text{ s}$$

Kinematic Procedure Four Square Part 2:

Student Practice Solution

Kinematic Procedure Four Square - Part 3

Create a Kinematic Diagram for each of the scenarios given.
Choose the appropriate equation and solve for the requested variable.

1. A car has an initial velocity of 12 m/s and coasts up a hill with an acceleration of -1.6 m/s^2 for 9.0 s. What is its displacement?	**2.** A car has a constant velocity 15 m/s and travels for 6.5 s? How far did it travel?
3. How far does a car that has an initial velocity of +10 m/s and accelerates to a final velocity of +15 m/s travel in 6.5 s?	**4.** How far does a plane travel while flying for 15 s as its velocity changes from 145 m/s to 75 m/s at a uniform rate of acceleration?

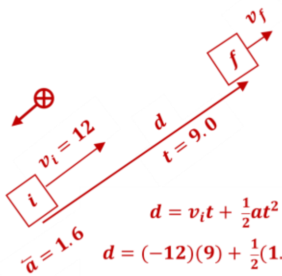
Kinematic Procedure Four Square Part 3:

Student Practice Blank

Kinematic Procedure Four Square - Part 3

Create a Kinematic Diagram for each of the scenarios given.
Choose the appropriate equation and solve for the requested variable.

1. A car has an initial velocity of 12 m/s and coasts up a hill with an acceleration of -1.6 m/s² for 9.0 s. What is its displacement?



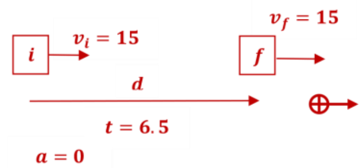
$d = v_i t + \frac{1}{2}at^2$

$d = (-12)(9) + \frac{1}{2}(1.6)9^2$

$d = -43.2 \text{ m}$

This is up the ramp, but as the car is sliding back down the ramp.

2. A car has a constant velocity 15 m/s and travels for 6.5 s? How far did it travel?

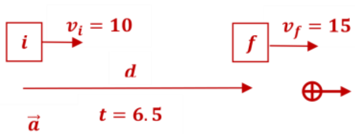


$d = \frac{1}{2}(v_f + v_i)t$

$d = \frac{1}{2}(15 + 15)(6.5)$

$d = 97.5 \text{ m}$

3. How far does a car that has an initial velocity of +10 m/s and accelerates to a final velocity of +15 m/s travel in 6.5 s?

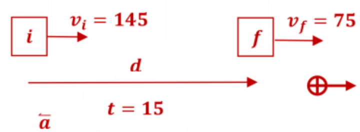


$d = \frac{1}{2}(v_f + v_i)t$

$d = \frac{1}{2}(15 + 10)6.5$

$d = 81.2 \text{ m}$

4. How far does a plane travel while flying for 15 s as its velocity changes from 145 m/s to 75 m/s at a uniform rate of acceleration?



$d = \frac{1}{2}(v_f + v_i)t$

$d = \frac{1}{2}(75 + 145)15$

$d = 1650 \text{ m}$

Kinematic Procedure Four Square Part 3:

Student Practice Solution

Additional Examples *Sans* Descriptions

It can be instructive to solve a kinematic problem based only on the kinematic diagram. Although a physical description can give students focus and comfort from familiarity of a physical system, it can also obscure some mathematical issues of which they should be aware and with which they are uncomfortable.

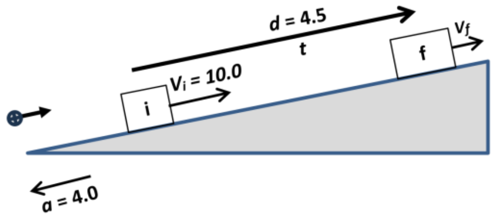
In particular two of the kinematic equations are quadratic, one in the velocity variables and the other in the time variable. In addition, interesting issues arise when the acceleration and displacement are oppositely directed.

It also emphasizes the universality of this solution method. The physical story behind a word problem often makes each problem seemingly different from all the others – that each problem on a four-square requires a unique approach and a separate method of solution. In fact, the solution to any (single acceleration, single object) kinematic problem is the same – a well-defined kinematic diagram which feeds into a single equation.

These cases are explored in the following few student additional worksheets *sans* descriptions.

Additional Kinematics - Part 1

For the kinematic diagram given at right:



a) Solve for the time *t*

b) Explain the physical reason for two different times.

c) Determine the final velocities when *d* = 4.5 m

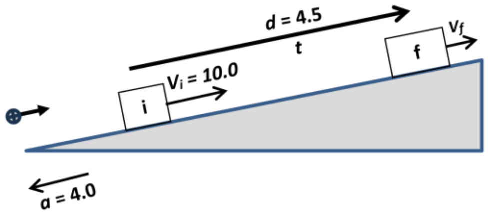
Additional Kinematics Problems Part 1:

Student Practice Blank

Additional Kinematics - Part 1

For the kinematic diagram given at right:

a) Solve for the time *t*



$$d = v_i t + \frac{1}{2}at^2$$

$$4.5 = (10)t - \frac{1}{2}(4)t^2$$

$$t = 0.5 \text{ s and } 4.5 \text{ s}$$

b) Explain the physical reason for two different times.

The mass slides up the ramp as it slows down, reaches the highest point, then slides back down. There are two times at which the mass is at the same position – once on the way up, and once on the way down.

c) Determine the final velocities when *d* = 4.5 m

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 10^2 + 2(4.0)(4.5)$$

$$v_f = \pm 11.67 \text{ m/s}$$

The situation above will occur whenever the direction of the acceleration and the initial velocity are in opposite directions.

This could be when an object is thrown upwards, an object that is initially sliding up an incline (in which case the ramp need be frictionless, otherwise the acceleration while moving up the ramp will be different than the acceleration when it slides down the ramp), or a spacecraft.

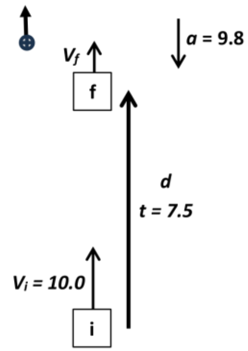
Additional Kinematics Problems Part 1:

Student Practice Solution

Additional Kinematics - Part 2

For the kinematic diagram at right:

a) **Find the displacement *d***



a) **For the given kinematic diagram above, solve for the final velocity v_f.**

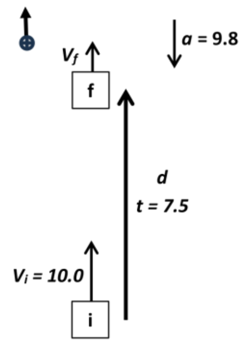
Additional Kinematics Problems Part 2:

Student Practice Blank

Additional Kinematics - Part 2

For the kinematic diagram at right:

a) **Find the displacement *d***



$$d = v_i t + \frac{1}{2}at^2$$

$$d = (10)(7.5) - \frac{1}{2}(9.8)(7.5)^2$$

$$d = -200\ \text{m}$$

a) **For the given kinematic diagram above, solve for the final velocity v_f.**

$$v_f = v_i + at$$

$$v_f = 10 - (9.8)(7.5)$$

$$v_f = -63\ \text{m/s}$$

The kinematic diagram here was purposefully drawn to maybe create cognitive dissonance within a student. If the problem read that a rock was thrown upwards at 10 m/s and you were to find the final velocity or the displacement after 7.5 seconds, it would not be unreasonable to have drawn the given kinematic diagram.

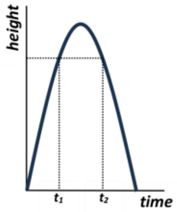
The diagram is not wrong – the choice for the displacement was chosen to be upwards and therefore with a decreased speed for v_f. The fact that the final velocity and displacement are calculated to be negative, is "correcting" the upward choice of direction of *d* and v_f, as well as a final speed that is larger than the initial speed. In reality the ball could have been thrown upwards from the top of a cliff, it slows as it rises, then falls downwards past the edge of the cliff from where it was released.

Additional Kinematics Problems Part 2:

Student Practice Solution

Additional Kinematics - Part 3

A rock is thrown straight upwards. It rises, slowing to zero speed, and then falls to the ground below. The height of the rock as a function of time is shown in the graph at right. At times t_1 and t_2, the rock is at a height of 8.5 m above the ground, and the acceleration of the rock is 9.8 m/s^2.



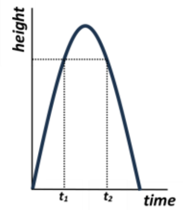
If the speed of the rock at time t_1 is 12.2 m/s, what is the speed of the rock at t_2?

Additional Kinematics Problems Part 3:

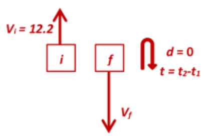
Student Practice Blank

Additional Kinematics - Part 3

A rock is thrown straight upwards. It rises, slowing to zero speed, and then falls to the ground below. The height of the rock as a function of time is shown in the graph at right. At times t_1 and t_2, the rock is at a height of 8.5 m above the ground, and the acceleration of the rock is 9.8 m/s^2.



If the speed of the rock at time t_1 is 12.2 m/s, what is the speed of the rock at t_2?



$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = (12.2)^2 + 2(9.8)(0)$$

$$v_f = 12.2\ \mathrm{m/s}$$

Since the speed is asked for, rather than the velocity, we need take the positive square root value only.

The initial and final positions, in fact, are the same point. This is 1-D motion vertically – there is no position in the horizontal. The boxes have been shifted to be able to see them separately. The U-turn arrow gives a place to anchor the displacement and time information. The actual displacement vector has zero length.

For a given trajectory of an object in free fall, two points at the same height will have the same speed. This will often be useful in projectile motion problems. For example, for a soccer ball that has been kicked into the air, the initial vertical speed of the ball when it is kicked is equal to the vertical speed of the ball when it strikes the ground on the way down.

Additional Kinematics Problems Part 3:

Student Practice Solution

Advanced Kinematic Problems

More advanced kinematic problems involve multiple objects that need to be considered, or an object that undergoes piecewise constant acceleration, or a combination of these.

For an object undergoing piecewise constant acceleration, we can stack the base kinematic diagrams so that the final velocity of the first segment becomes the initial velocity of the second segment.

For multiple objects, we can stack the base kinematic diagrams so that we can see the positional and temporal relationships between the two diagrams.

The “solution” for each base kinematic diagram is then typically an equation that may involve several unknowns. In problems with multiple objects, an equation relating the times and an equation relating the displacements are also possible. We therefore end up with a set of simultaneous equations to solve.

Road Barrier

The driver of a car suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75 s to begin applying the brakes during which time the car moves with a constant speed of ~~90.0 km/hr~~. During the braking, assume a constant acceleration of 10.0 m/s^2. Determine if the car hits the barrier or not?

90 km/hr = 25 m/s

Advanced Kinematics Problems:

Road Barrier

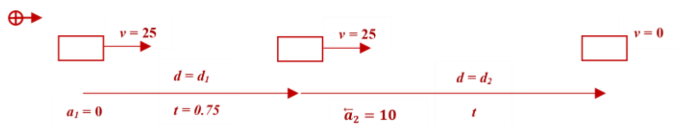
Student Practice Blank

Road Barrier

The driver of a car suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75 s to begin applying the brakes during which time the car moves with a constant speed of ~~90.0 km/hr~~. During the braking, assume a constant acceleration of 10.0 m/s^2. Determine if the car hits the barrier or not?

90 km/hr = 25 m/s

Solution 1 – How far does it take to stop?



$$d = v_i t$$

$$d_1 = (25)(0.75)$$

$$d_1 = 18.75$$

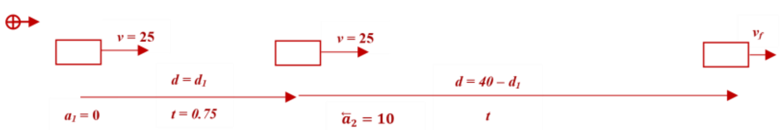
$$v_f^2 = v_i^2 + 2ad$$

$$0 = 25^2 - 2(10)d_2$$

$$d_2 = 31.25$$

Total distance needed to stop = $d_1 + d_2$ = 50 m
Total distance is greater than the 40 m to the barrier. CRASH!

Solution 2 – Is there still a positive velocity after 40 m?



$$d = v_i t$$

$$d_1 = (25)(0.75)$$

$$d_1 = 18.75$$

$$d_1 + d_2 = 40 \;\; so \;\; d_2 = 21.25$$

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 25^2 - 2(10)(21.25)$$

$$v_f = 14.1\ m/s$$

After 40 m, the velocity is still 14.1 m/s.
It is still moving toward the barrier. CRASH!

Advanced Kinematics Problems:

Road Barrier

Student Practice Solution

Road Barrier - Extension

The driver of a car suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75 s to begin applying the brakes during which time the car moves with a constant speed V_o. During the braking, assume a constant acceleration of 10.0 m/s^2. Determine the maximum speed V_o the car can have without crashing into the barrier.

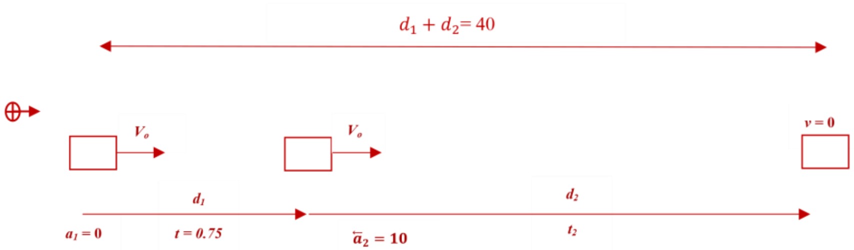
Advanced Kinematics Problems:

Road Barrier Extension

Student Practice Blank

Road Barrier - Extension

The driver of a car suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75 s to begin applying the brakes during which time the car moves with a constant speed $\boldsymbol{V_o}$. During the braking, assume a constant acceleration of 10.0 m/s^2. Determine the maximum speed $\boldsymbol{V_o}$ the car can have without crashing into the barrier.



$$d = v_i t$$

$$d_1 = V_o(0.75)$$

$$v_f^2 = v_i^2 + 2ad$$

$$0 = {V_o}^2 - 2(10)d_2$$

$$d_2 = \frac{{V_o}^2}{20}$$

$$d_1 + d_2 = 40$$

$$V_o(0.75) + \frac{V^2}{20} = 40$$

$${V_o}^2 + 15V_o - 800 = 0$$

$$V_o = 21.76 \text{ m/s}$$

Advanced Kinematics Problems:

Road Barrier Extension

Student Practice Solution

Hooptie-Police Race

Suppose that you are in a hurry to get home to fetch your physics lab that you forgot to bring to school this morning and that you had worked on ever so carefully to get that elusive A+. You try to get there as fast as possible, so you go as fast as your Hooptie can go – 80 mph.

Suddenly you pass a police car that, starting from rest, starts to give chase just as you pass. The police car, which has a most impressive engine, can accelerate to 120 mph in 10 seconds, and has a top speed of 120 mph. How much time does it take for the police car to catch up to your speeding Hooptie?

Advanced Kinematics Problems:

Hooptie-Police Race

Student Practice Blank

Hooptie-Police Race

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P $v = 0$ — $d = d_1$, $\vec{a_1}$, $t_1 = 10$ — P $v = 120$ — $d = d_2$, $a_2 = 0$, $t2$ — P $v = 120$

$d = \frac{1}{2}(v_f + v_i)t$

$d_1 = \frac{1}{2}(0 + 120)10$

$d_1 = 600\,\frac{mi}{hr} \cdot s$

$d = v_i t + \frac{1}{2}at^2$

$d_2 = 120\,\frac{mi}{hr}\,t_2$

⊕→

P $v = 80$ — d_H, $a_H = 0$, t_H — P $v = 80$

$d = v_i t + \frac{1}{2}at^2$

$d_H = 80\,\frac{mi}{hr}\,t_H$

The 3 kinematic diagrams give us 3 equations, but we have four unknowns.

By aligning the kinematic diagrams to reflect that the Hooptie and the Police cars are both at the same position and time initially and finally, we also have two additional equations that we can used just based on this time and position restraint:

$t_1 + t_2 = t_H$

$t_2 = t_H - 10\ s$

$d_1 + d_2 = d_H$

$600\,\frac{mi}{hr} \cdot s + 120\,\frac{mi}{hr}\,t_2 = 80\,\frac{mi}{hr}\,t_H$

$600\,\frac{mi}{hr} \cdot s + 120\,\frac{mi}{hr}(t_H - 10\ s) = 80\,\frac{mi}{hr}\,t_H$

$t_H = 15\ s$

It is good practice to make sure all units are standard, or at least consistent. However, sometimes, if we forgo the conversions until the end, we may not have to convert at all as in this case, or only once at the very end.

Advanced Kinematics Problems:

Hooptie-Police Race

Student Practice Solution

Cheetah-Gazelle Chase

As most children know, the cheetah is the fastest land animal, but it cannot run fast for very long. Suppose a cheetah can accelerate from 0 to 100 km/hr in 5 s (assume constant acceleration) and then can maintain 100 km/hr for 10 s after that. [After 15 s, the cheetah cannot continue and stops running.]

A very hungry cheetah, starting from rest, chases after a gazelle which can run at 50 km/hr (assume constant velocity). From what maximum distance must the cheetah begin the chase in order to catch its prey? [Assume the gazelle has no acceleration period – that is, its speed is 50 km/hr the entire time.]

Advanced Kinematics Problems:

Cheetah-Gazelle Chase

Student Practice Blank

Cheetah-Gazelle Chase

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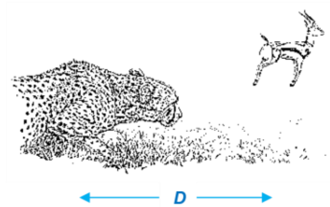
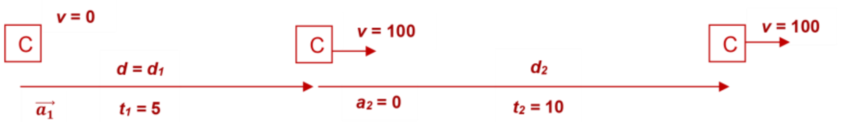




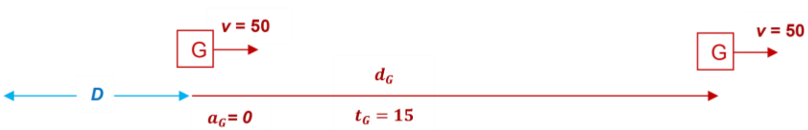
$$d = \frac{1}{2}(v_f + v_i)t$$

$$d_1 = \frac{1}{2}(0 + 100)5$$

$$d_1 = 250\,\frac{km}{hr}\cdot s$$

$$d = v_i t + \frac{1}{2}at^2$$

$$d_2 = 1000\,\frac{km}{hr}\cdot s$$



$$d = v_i t + \frac{1}{2}at^2$$

$$d_G = 750\,\frac{km}{hr}\cdot s$$

By aligning the kinematic diagrams to reflect that the Gazelle and the Cheetah are both at the same position and time after 15 s, but initially are separated by D, we can "see" the relationship between the displacements.

$$d_1 + d_2 = D + d_G$$

$$250\,\frac{km}{hr}\cdot s + 1000\,\frac{km}{hr}\cdot s = D + 750\,\frac{km}{hr}\cdot s$$

$$D = 500\,\frac{km}{hr}\cdot s$$

$$D = 0.139\ km$$

It is good practice to make sure all units are standard, or at least consistent.
However, sometimes, if we forgo the conversions until the end, we may not have to convert at all, or as in this case, only once.

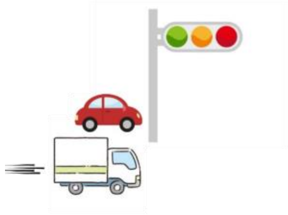
Advanced Kinematics Problems:

Cheetah-Gazelle Chase

Student Practice Solution

Car-Truck Race

A car is at rest at a stop light. When a truck traveling at a constant speed of $v = 15$ m/s reaches the car, the light turns green and the car accelerates with an acceleration $a = 2.5$ m/s^2.



a) How long does it take for the car to catch the truck? 12 s

b) How far do they both travel during this time? 180 m

Advanced Kinematics Problems:

Car-Truck Race

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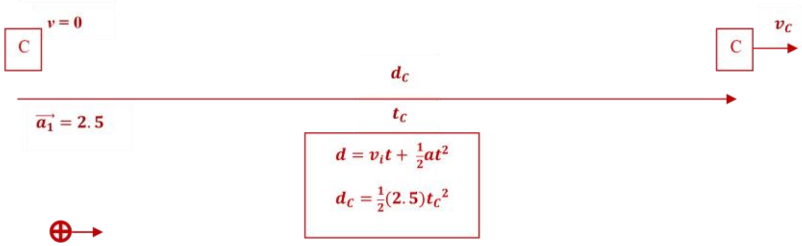

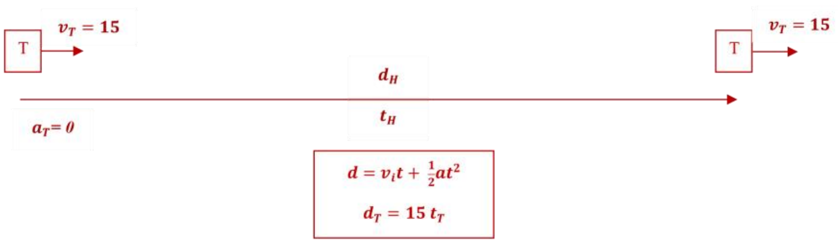

The truck and the car are both at the same initial and final positions at the same initial and final times. Therefore:

$$d_C = d_T \equiv D \quad \text{and} \quad t_C = t_T \equiv T$$

which gives us two simultaneous equations:

$$D = 1.25T^2$$
$$D = 15\,T$$

with the solution:

T = 12 s and D = 180 m

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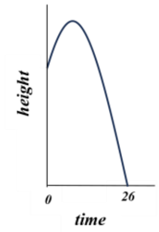
Rocket

A rocket is launched straight up from the ground. At some point during its ascent its engine fails. The rocket strikes the ground with a speed of 160 m/s 26 seconds after the engine fails. A sketch of the height of the rocket as a function n of time after the engine fails is presented to the right.

a) What was the speed of the rocket when the engine failed?

b) What was the maximum height above the ground that the booster reached?



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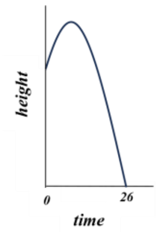

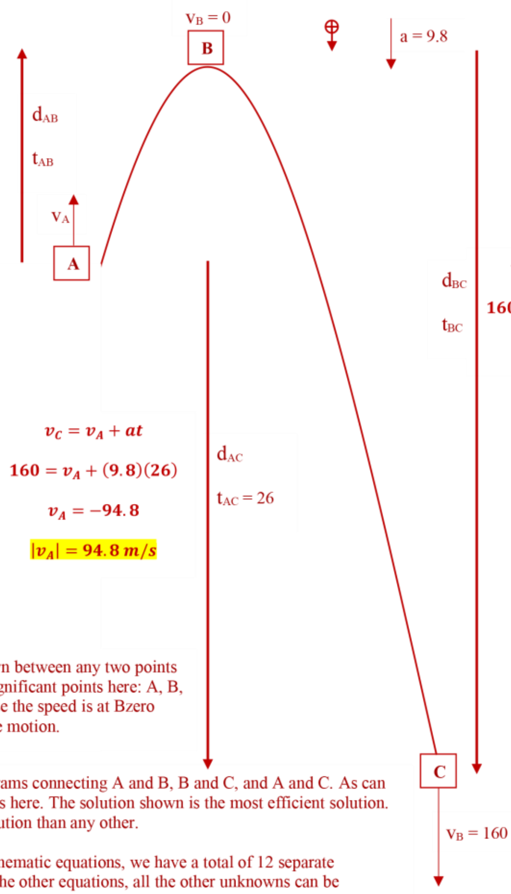

$$v_C^2 = v_B^2 + 2ad_{BC}$$

$$160^2 = 0^2 + 2(9.8)d_{BC}$$

$$d_{BC} = 1310\ m$$

$$v_C = v_A + at$$

$$160 = v_A + (9.8)(26)$$

$$v_A = -94.8$$

$$|v_A| = 94.8\ m/s$$

Kinematic diagrams can be drawn between any two points of the motion. There are three significant points here: A, B, and C. [Point B is special because the speed is at Bzero since it is the highest point of the motion.

We can construct kinematic diagrams connecting A and B, B and C, and A and C. As can be seen, there are many unknowns here. The solution shown is the most efficient solution. This does not make it a better solution than any other.

Since each diagram supports 4 kinematic equations, we have a total of 12 separate equations at our disposal. Using the other equations, all the other unknowns can be determined.

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